МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ

Центральноукраїнський державний університет

імені Володимира Винниченка

Т. А. Капітан, Т. О. Лелека

ENGLISH FOR MATHEMATICIANS

Навчально-методичний посібник

з курсу «Іноземна мова (англійська) за професійним спрямуванням»

для студентів факультету математики, природничих наук та технологій

Кропивницький - 2023

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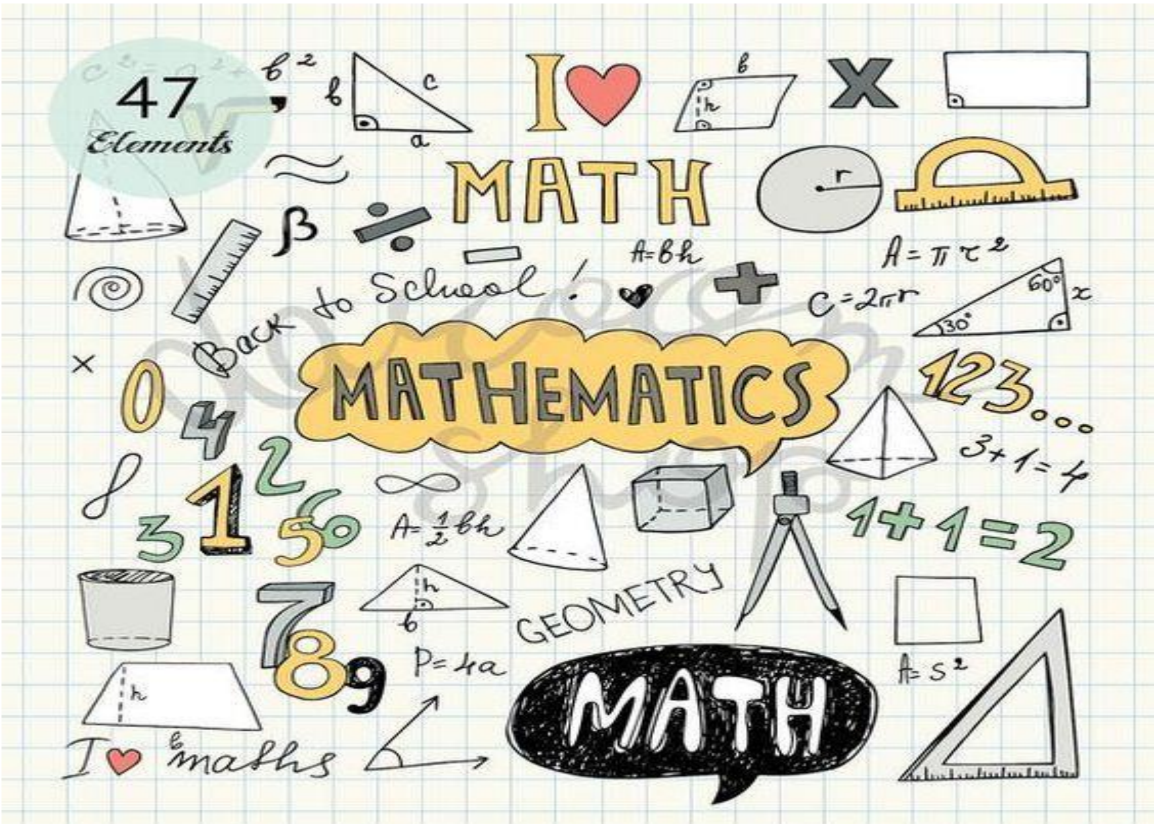

Кропивницький – 2023

УДК 811.111(072)

К 20

Укладачі: к.ф.н., доц. Капітан Т.А., к.ф.н., доц Лелека Т.О. ENGLISH FOR MATHEMATICIANS Навчально-методичний посібник з курсу «Іноземна мова (англійська) за професійним спрямуванням» для студентів факультету математики, природничих наук та технологій. – Кропивницький: ФОП Піскова М. А., 2023. – 224 с.

Рецензенти:

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Упровадження навчально-методичного посібника з іноземної мови (англійської) за професійним спрямуванням передбачено для студентів факультету математики, природничих наук та технологій. У змісті посібника подано десять тем, у яких студентам запропоновано матеріали для вдосконалення усного мовлення, читання, письма. В опрацюванні кожної теми передбачено також індивідуальну роботу. Структура всіх тем однотипна. Лаконічність і доступність наведеного матеріалу дає змогу швидко, без зайвих зусиль усвідомити специфіку кожного завдання. Матеріал, викладений у посібнику, дозволяє активізувати пізнавальну діяльність студентів у процесі вивчення дисципліни; мотивувати їх до підвищення рівня володіння фаховою іноземною мовою.

Друкується за рішенням методичної ради

Центральноукраїнського державного університету

імені Володимира Винниченка

Протокол № 3 від 17 травня 2023 року.

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ПЕРЕДМОВА

Навчально-методичний посібник розроблено відповідно до програми навчальної дисципліни «Іноземна мова» для студентів закладів вищої освіти, які здобувають підготовку за освітньо-кваліфікаційним рівнем бакалавра Спеціальність 014.04 Середня освіта (Математика) Освітня програма Середня освіта (Математика та Інформатика) Середня освіта (Математика, Економіка).

Упровадження навчально-методичного посібника з іноземної мови (англійської) за професійним спрямуванням передбачено для студентів факультету математики, природничих наук та технологій.

Мета пропонованої праці – забезпечити засвоєння студентами водночас із загальновживаною лексикою й повсякденними розмовними виразами, які, безумовно, важливі для фахівців будь-якої галузі, також спеціальної лексики; вироблення навичок читання фахових текстів; підвищення рівня усного та писемного мовлення завдяки повторенню граматичних структур і моделей.

Матеріали посібника викладено згідно з програмою вивчення іноземної мови у ЗВО і вимогами кваліфікаційного рівня. У змісті навчально-методичного посібника подано десять тем, у яких студентам запропоновано матеріали для вдосконалення усного мовлення, читання, письма. В опрацюванні кожної теми передбачено **індивідуальну роботу.** Структура всіх тем однотипна, що дозволяє студентам планувати свій робочий час. Лаконічність і доступність наведеного матеріалу дає змогу швидко, без зайвих зусиль усвідомити специфіку кожного завдання. Матеріал, викладений у посібнику, дозволяє активізувати пізнавальну діяльність студентів у процесі вивчення дисципліни; мотивувати їх до підвищення рівня володіння фаховою іноземною мовою; вивільнити час для практичної підготовки.

UNIT 1 MATHEMATICAL SCHOOLS

Exercise 1. Read and memorise the following words and word combinations. differentiation – диференціювання, відшукання похідної

integration – інтегрування, обчислення інтеграла

measure – міра; показник; критерій; масштаб

делитель infinite series – нескінченний ряд

calculus – 1) обчислення 2) математичний аналіз (навчальна дисципліна, розділ найвищої математики)

mathematical object – математичний об'єкт

topological space – топологічний простір

metrical space – метричний простір

method of exhaustion – метод послідовних елімінацій

regular polygon – правильний багатокутник

limit – 1) межа; межа 2) pl. інтервал значень

infinitesimal [ɪnfɪnɪ'tesɪm(ə)l] – нескінченно мала величина

infinitesimal calculus – аналіз нескінченно малих величин

sine [saɪn], cosine ['kəusaɪn], tangent ['tænʤ(ə)nt], arctangent – синус, косинус, тангенс, арктангенс

derivative – похідна, похідна функція

the distributive law – розподільний (дистрибутивний) закон

to combine together – поєднувати, об'єднувати, комбінувати

the law of combination – закон комбінацій (поєднання)

a scalar product – скалярний добуток

a progression of abstractions – послідовність абстракцій

to belong to the group – належати групі

to make no difference – не мати значення

Exercise 2. Write the transcription and translate the following words. Make sentences with them.

abstruct number

a four – figure number

concrete number

cardinal number

ordinal number

posiтive number

negative number

algebraic sуmbol

mixed number

wнole number (integer)

fraction

even numbers

odd number

prime number

complex number

real

imaginary pa

proper fraction

numerator

denominator

improrer fraction

Exercise 3. Read and translate the text

Great minds of Greece such as Thales, Pythagoras, Euclid, Archimede, Apollonius, Eudoxus, etc. produced an amazing amount of first class mathematics. The fame of these mathematicians spread to all corners of the Mediterranean world and attracted numerous pupils. Masters and pupils gathered in schools which though they had few, buildings and no campus were truly centers of learning. The teaching of these schools dominated the entire life of the Greeks. Despite the unquestioned influence of Egypt and Babylonia on Greek mathematicians, the mathematics produced by the Greeks differed fundamentally from that which preceded it. It was the Greeks who founded mathematics as a scientific discipline. The Pythagorean School was the most influential in determining both the nature and content Greek mathematics. Its leader Pythagoras founded a community which embraced both mystical and rational doctrines. The original Pythagorean brotherhood (c. 550—300 B. C.) was a secret aristocratic society whose members preferred to operate from behind the scenes and, from there, to rule social and intellectual affairs with an iron hand. Their noble born initiates were taught entirely by word of mouth. Written documentation was not permitted, since anything written might give away the secrets largely responsible for their power.

Among these early Pythagoreans were men who knew more about mathematics then available than most other people of their time. They recognized that vastly superior in design and manageability Babylonian base-ten positional numeration system might make computational skills available to people in all walks of life and rapidly democratize mathematics and diminish their power over the masses. They used their own non positional numeration system (standard Greek alphabet supplemented by special symbols).

Although there was no difficulty in determining when the symbols represented a number instead of a word, for computation the people of the lower classes had to consult an exclusive group of experts or to use complicated tables and both of these sources of help were controlled by the brotherhood. For Pythagoras and his followers the fundamental studies were geometry, arithmetic, music, and astronomy. The basic element of all these studies was number not in its practical computational aspect, but as the very essence of their being; they meant that the nature of numbers should be conceived with the mind only. In spite of the mystical nature of much of the Pythagorean study the members of community contributed during the two hundred or so years following the founding of their organization, a good deal of sound mathematics.

Thus, in geometry they developed the properties of parallel lines and used them to prove that "the sum of the angles of any triangle is equal to two right angles". They contributed in a noteworthy manner to Greek geometrical algebra, and they developed a fairly complete theory of proportional though it was limited to commensurable magnitudes, and used it to deduce properties of similar figures. They were aware of the existence of at least three of the regular polyhedral solids, and they discovered the incommensurability of a side and a diagonal of a square.

Details concerning the discovery of the existence of incommensurable quantities are lacking, but it is apparent that the Pythagoreans found it as difficult to accept incommensurable quantities as to discover them. Two segments are commensurable if there is a segment that "measures" each of them – that is, it contains exactly a whole number of times in each of the segments.

Exercise 4. Which of these statements are true? Correct the false ones.

1. It was the Egyptians who founded mathematics as a scientific discipline. 2. The Pythagorean School was the most influential in determining both the nature and content Greek mathematics. 3. There was some difficulty in determining when the symbols represented a number instead of a word. 4. For Pythagoras the fundamental studies were geometry, arithmetic, music, and astronomy. 5. Two segments are commensurable if there is a segment that "measures" each of them.

Exercise 5. Find derivates in the text for the following words. Explain their meaning in Ukrainian:

Verb	Noun	Adjective
to symbolize		Symbolic
to differ	Difference	
	Supposition	Suppositional
to present		Representational
to compact		Compact

Exercise 6. Translate from Ukrainian into English.

У галузі математики, як алгебра, невідомі величини виражаються буквами. Деякі літери називаються змінними, оскільки числа, які вони представляють, змінюються від рівняння до рівняння. Інші літери називаються константами, тому що ними представлені числа, що мають постійну величину, яка ніколи не змінюється.

Exercise 7. Translate the following phrases into mathematical expressions or equations.

1. Six less than twice a number is forty five. 2. A number minus seven yields ten 3. A total of six and some number 4. Twelve added to a number 5. Eight times a number is forty-eight 6. The produce of fourteen and a number

Exercise 8. Match these terms with their definitions.

1. notation	a) not representing any specific value
2. arbitrary	b) having all sides of equal length
3. equilateral	c) any series of signs or symbols used to represent quantities or elements in a specialized system
4. a solid	d) a solid figure having four plane faces
5. tetrahedron	e) a solid figure having eight plane faces
6. octahedron	f) a closed surface in three-dimensional space
7. vertex (pl.vertices)	g) the point of intersection of two sides of a plane figure or angle
8. an obtuse angle	h) (of a triangle) having two sides of equal length
9 .an isosceles triangle	j) (of an angle) lying between 90° and 180°

Exercise 9. Read and write the numbers and symbols in full according to the way they are pronounced:

76, 13, 89, 53, 26, 12, 11, 71, 324, 117, 292, 113, 119; 926, 929, 735, 473, 1002, 1026, 2606, 7354, 7013, 3005, 10117, 13526, 17427, 72568, 634113, 815005, 905027, 65347005, 900000001, 10725514, 13421926, 65409834, 815432789, 76509856.

Exercise 10. Form adjectives from the following verbs by adding suffix- "able". Translate them into Ukrainian. Example: comfort-comfortable

1. rely 2. desire 3. read 4. explain 5. manage 6. use 7. forget 8. predict 9. measure 10. remove 11. renew 12. imagine 13. agree 14. enjoy. 15. depend 16 consider 17. prove 18. computer.

Exercise 11. Form adjectives from the following verbs by adding suffix - "ive". Translate them into Ukrainian.

Example: express – expressive

1. collect 2. impress 3. conduct 4. act 5. oppress 6. effect 7. dominate 8. adapt 9. compress.

Exercise 12. Form nouns from the following adjectives by adding suffix – (i) "ty". Translate them into Ukrainian.

Example: odd – oddity; curious – curiosity.

1.diverse 2. hostile 3. complex 4. reliable 5. historic 6. active 7. formal 8. probable 9. topical 10. universal 11. certain 12. vital 13. modern 14. safe 15. prior 16. valid 17. severe 18. special 19. novel 20. mature 21. local 22. possible 23. able 24. capable 25. noble 26. obscure 27. modal 28. adaptive 29. conductive 30. passive. 31. computable 32. tractable 33. applicable 34. clear 35. lucid.

Exercise 13. Read and translate the text.

Counting in the Early Ages Counting is the oldest of all processes. It goes back to the very dawn of human history. At all times and practically in all places, people had to think of supplies of food, clothing and shelter. There was often not enough food or other things. So, even the most primitive people were always forced to think of how many they were, how much food and clothing they possessed, and how long all those things would last.

These questions could be answered only by counting and measuring. How did people count in the dim and distant past, especially when they spoke different languages? Suppose you wanted to buy a chicken from some poor savage tribe.

You might point toward some chickens and then hold up one finger. Or, instead of this, you might put one pebble or one stick on the ground.

At the same time, you might make a sound in your throat, something like ung, and the savages would understand that you wanted to buy one chicken. But suppose you wanted to buy two chickens or three bananas, what would you do? It would not be hard to make a sign for the number two. You could show two fingers or point to two shoes, to two pebbles, or to two sticks. For three you could use three fingers or three pebbles, or three sticks. You see that even though you and the savages could not talk to one another, you could easily make the numbers one, two, and three known. It is a curious fact that much of the story of the world begins right here.

Have you ever tried to imagine what the world would be like if no one had ever learned how to count or how to write numerals? We are so in the habit of using numbers that we rarely think of how important they are to us. For example, when we open our eyes in the morning, we are likely, first of all, to look at the clock, to see whether it is time to get up. But if people had never learned to count, there would be no clocks. We would know nothing of hours or minutes, or seconds. We could tell time only by the position of the sun or the moon in the sky; we could not know the exact time under the best conditions, and in stormy weather, we could only guess whether it was morning or noon, or night.

The clothes we wear, the houses we live in, and the food we eat, all would be different if people had not learned how to use numbers. We dress in the morning without stopping to think that the materials of which our clothing is made have been woven on machines adjusted to a fraction of an inch.

The number and height and width of the stair steps on which we walk were carefully calculated before the house was built. In preparing breakfast, we measure so many cups of cereal to so many cups of water; we count the minutes it takes to boil the eggs, or make the coffee. When we leave the house, we take money for bus fare unless we walk and for lunch unless we take it with us; but if people could not count, there would be no money. All day long, we either use numbers ourselves or use things that other people have made by using numbers.

It has taken people thousands of years to learn how to use numbers, or the written figures, which we call numerals. For a long time after men began to be civilized, such simple numbers as two and three were all they needed. For larger numbers, they used words in their various languages which correspond to expressions, such as lots of people, a heap of apples, a school of fish, and a flock of sheep.

For example, a study of thirty Australian languages showed no number above four, and in many of these languages there were number names for only one and two, the larger numbers being expressed simply as much and many. You must have heard about the numerals, or number figures, called digits.

The Latin word digiti means fingers. Because we have five fingers on each hand, people began, after many centuries, to count by fives. Later, they started counting by tens, using the fingers of both hands. Because we have ten toes as well as ten fingers, people counted fingers and toes together and used a number scale of twenty. In the English language, the sentence “The days of a man’s life are three score years and ten” the word score means twenty (so, the life span of humans was considered to be seventy). Number names were among the first words used when people began to speak.

The numbers from one to ten sound alike in many languages. The name digits was first applied to the eight numerals from 2 to 9. Nowadays, however, the first ten numerals, beginning with 0, are usually called the digits. It took people thousands of years to learn to write numbers, and it took them a long time to begin using signs for the numbers; for example, to use the numeral 2 instead of the word two.

When people began to trade and live in prosperous cities, they felt a need for large numbers. So, they made up a set of numerals by which they could express numbers of different values, up to hundreds of thousands. People invented number symbols. To express the number one, they used a numeral like our 1. This numeral, probably, came from the lifted finger, which is the easiest way of showing that we mean one. The numerals we use nowadays are known as Arabic. But they have never been used by the Arabs. They came to us through a book on arithmetic which was written in India about twelve hundred years ago and translated into Arabic soon afterward. By chance, this book was carried by merchants to Europe, and there it was translated from Arabic into Latin. This was hundreds of years before books were first printed in Europe, and this arithmetic book was known only in manuscript form.

Exercise 14. Match the following.

1. distant past	a) визначати час
2. to tell time	b) далеке минуле
3. to perform operations	c) точний час
4. exact time	d) виконувати операції
5. rarely	e) винаходити
6. to invent	f) рідко
7. digit	g) зберігати інформацію
8. to store information	h) однозначне число
9. to record	i) пристосування
10. device	j) записувати
11. ancient times	k) процвітати
12. to prosper	l) давні часи
13. abacus	m) лічильна дошка
14. to print	n) рахунки
15. counting table	o) друкувати

Exercise 15. Discussion point.

What if you were to give a short presentation on Euclid's most important contribution to mathematics? What would you say?

Exercise 16. Complete these sentences by putting the verb in brackets into the Present Simple or the Present Continuous.

To solve the problem of gravitation, scientists …………… (consider) time-space geometry in a new way nowadays. Quantum rules …………… (obey) in any system. We …………… (use) Active Server for this project because it ……… (be) Webbased. Scientists …………… (trace and locate) the subtle penetration of quantum effects into a completely classical domain. Commonly we ………… (use) C + + and JavaScript. At the moment we …………… (develop) a Web–based project. Its domain ………… (begin) in the nucleus and ………… (extend) to the solar system. Right now I ………… (try) to learn how to use Active Server properly.

Exercise 17. Put "can", "can not","could", "could not" into the following sentences.

Parents are finding that they ………….. no longer help their children with their arithmetic homework. The solution for the construction problems …………… be found by pure reason. The Greeks …………….. solve the problem notbecause they were not clever enough, but because the problem is insoluble under the specified conditions. Using only a straight-edge and a compass the Greeks …………. easily divide any line segment into any number of equal parts. Web pages…………. offer access to a world of information about and exchange with other cultures and communities and experts in every field.

Exercise 18. Fill in each blank with an appropriate preposition: of, to, in, at, through, with, on. One preposition can be used several times.

… our modern world, mathematics is related … a very large number … important human activities. Make a trip … any modern city, look …the big houses, plants, laboratories, museums, libraries, hospitals and shops, … the system … transportation and communication. You can see that there is practically nothing … our modern life which is not based … mathematical calculations. … co-operation … science, mathematics made possible our big buildings, railroads, automobiles, airplanes, spaceships, subways and bridges, artificial human organs, surgical operations and means of communication that in the past seemed fantastic and could never be dreamt … .

Exercise 19. Use the correct tense / voice form of the verb.

Model: A lot of knowledge (to accumulate) in the second half of the 20th century. A lot of knowledge was accumulated in the second half of the 20th century.

1. In the early ages, primitive counting (to do) with the help of gestures, objects, fingers and toes. 2. The work of Leibniz (to publish) several years before Newton's results appeared in print. 3. In the past, people could not foresee that their life (to change) radically due to technological advances. 4. Scientists (to make) their discoveries due to the achievements of their predecessors. 5. Mathematics (to be) a science of numbers before it became a science of relations. 6. Archimedes (to make) his discovery while (to take a bath). 7. All spheres of life (to benefit) from computers in the future. 8. Many problems of artificial intelligence (not to solve) yet. 9. A lot of useful gadgets (to appear) in the last 10 years. 10. Nowadays, science and technology (to develop) at a great speed. 11. It is believed that in the future computers (to make) people's life still more comfortable. 12. Mathematics (to contribute) the most to the development of computer science. 13. Without the computer, the present day achievements of many sciences (to be) impossible. 14. Very little (to know) to us about the life of Euclid. 15. Einstein (to be) young when he developed the theory of relativity. 16. Lobachevsky's new idea (to remain) unnoticed for a long time. 17. Till his dying day, Galileo was true to his ideas, though he (to renounce) them before under the pressure of the Inquisition. 18. Some new branches of mathematics (to develop) in the 20th century. 19. It (to take) mathematicians over three hundred years to prove Fermat's last theorem. 20. Mathematical language (to characterize) by its symbolic nature, brevity and precision.

Exercise 20. Ask special questions using the words in parentheses.

1. There are really two types of problems involved here. (How many?) 2. Having understood the ideas, we can simplify our notation. (When?) 3. Being interested in set theory, he never missed his special course. (Why?) 4. Rational functions are functions involving an additional operation of division. (What?) 5. A

point representing a variable is called a variable point. (How?) 6. The students studying the theory of sets find this statement interesting. (Who?) 7. Equations containing one or more variables to the first power only are linear in one or two variables. (What?) 8. We can find that some elements form a smaller group inside the big one. (What?) 9. Groups can arise in many quite distinct situations. (In what cases?) 10. When speaking of quantities, we shall have in view their numerical values. (What?) 11. The meanings of these words are often confused in speech. (Where?)

Exercise 21. Read, translate and give a short outline of the text in English:

Many thousands of years ago this was a world without numbers. Nobody missed them. Primitive men knew only ten number-sounds. The reason was that they counted in the way a small child counts today, one by one, making use of their fingers. The needs and possessions of primitive men were few: they required no large numbers. When they wanted to express a number greater than ten they simply combined certain of the ten sounds connected with their fingers. Thus, if they wanted to express "one more than ten" they said "one-ten" and so on. Nowadays Maths has become an inseparable part of our lives and whether we work in an office or spend most of our time at home, each one of us uses Maths as a part of our everyday life. No matter where we are as well as whatever we are doing, Maths is always there whether you notice it or not. When you buy a car, follow a recipe, or decorate your home, you are using Maths principles. People have been using these same principles for thousands of years, across countries and continents. Whether you are sailing a boat off the coast of Japan or building a house in Peru, you are using Maths to get things done. How can Maths be so universal? First, human beings did not invent Maths concepts; we discovered them. Also, the language of Maths is numbers, not English or German or Russian. If we are well-versed in this language of numbers, it can help us make important decisions and perform everyday tasks. Maths can help us to shop wisely, buy the right insurance, remodel a home within a budget, understand population growth, or even bet on the horse with the best chance of winning the race.

Exercise 22. Fill in the gaps with the words from Text

1. Many thousands of years ago this was a world without _______. 2. Primitive men counted making use of _______. 3. The possessions of primitive men _______ no large numbers. 4. When they wanted to _______ they simply combined the sounds connected with their fingers. 5. Nowadays Maths has become an ________ of our lives. 6. We use _______ when you buy a car, follow a recipe, or decorate your home. 7. How can Maths be so _______? 8. The reason is we did not _______ Maths concepts; we _______ them. 9. We should be _______ in the language of numbers. 10. Maths can help us make important _______ and _______ everyday tasks.

Exercise 23. Translate into English

Десятична система нумерації виникла Індії. Згодом її стали називати «арабською», бо її було перенесено до Європи арабами. Цифри, якими ми користуємося, теж називаються арабськими. У цій системі особливо важливе значення має десять, і тому система зветься десятковою системою нумерації. Щоб легше читати багатозначні числа, ми відокремлюємо цифри в них комами по три групи. Групу із трьох цифр ми називаємо періодом.

Exercise 24. Read the numbers

48; 392; 712; 947; 2,364; 6,839; 12,578; 83,740; 267,394; 847,253; 2,746,938; 4,957,816; 34,689,256; 67,912,378; 546,873,923; 489,736,263.

Exercise 25. Read and translate the text

Modern mathematics has spread into some interesting and incredibly useful areas of modern life. Highway engineers use optimization techniques and linear algebra to analyze traffic patterns and minimize travel time for commuters. Airlines, hotels, and Broadway theaters (not to mention some retailers) use complex mathematical models, so complex that only computer programs can sometimes solve them, to set prices on a dynamic basis. Physicists use group theory and tensor analysis to solve the mysteries of sub-atomic particles. There are numerous other examples, from statistics being used by the Census Bureau and research firms to predict market trends, to partial differential equations being used by brokerage houses to formulate models of where the market is going.

Origins of Mathematics

But where did mathematics originate from? How did it get started, and how complicated (or simple) were its beginnings in the ancient world?

While arithmetic in some form (counting) has been with us since people banded together in primitive tribal groups 35,000 years ago, formal mathematics could not begin until writing was invented. This event occurred around 3200 BCE in the Fertile Crescent (specifically Mesopotamia, near the ancient city of Sumer); some authorities think it was also independently invented in ancient Egypt around the same time, as well as in China 1200 BCE. The ancient Egyptians may have made important contributions as well.

For example, the “Moscow mathematical papyrus” (so called because it is held in a Moscow museum and dates to 1850 BCE) contains a problem analyzing the dimensions of a truncated pyramid. While the hieroglyphic explanation would seem like any other Egyptian manuscript to modern eyes, the diagram accompanying it would be readily recognizable in any modern algebra class.

So what is the link between mathematics and writing, and why was it necessary for the development of writing to precede the development of a mathematics that could go beyond counting and simple arithmetic?

Besides the relative permanence of writing and the ability to transmit information from one generation to the next, writing is distinguished from other symbolic representational systems (e.g., cave art, or temple decorations) by the fact that its symbols (letters) are not related to their meaning, but rather represent sounds or other phonemes as the abstract building blocks of language. This development moves writing beyond short-hand pictorial representations, so that the story is told by putting together abstract letters to form words, and not by recounting a story in pictures.

Likewise, mathematics could not get started until the beginnings of a symbolic language to represent basic mathematical concepts had been developed. Arithmetic started roughly at the same time as writing began, around 4000 BCE in the Fertile Crescent (in what is now modern Iran), and at first was based only on counting techniques. For example, arithmetic at that time might be used to represent that 4 apples plus 5 apples yields 9 apples.

Mathematics as we understand it today, where math symbols take on abstract meanings beyond simple enumeration and arithmetic, did not really have its beginnings until the time of the ancient Greeks (eighth century BCE). The Pythagorean theorem (sixth century BCE), which says that the sum of the squares of the two legs of a right triangle is equal to the square of the hypotenuse, is generally thought to be the most ancient mathematical formulation to go beyond simple arithmetic and geometry. The Greeks are also credited with being the first to develop deductive logic, a type of reasoning fundamental to mathematics, whereby one can prove a theorem or statement to always be true.

Again, while the text would be incomprehensible to anyone today who did not read ancient Greek, the diagrams in the fragment pictured below are readily recognizable to any college (or high school) algebra student. Here is a fragment of Euclid's "Elements" found at Oxyrhynchus (in Egypt) and dating to roughly 100 CE.

Ancient Greek Contributions to Mathematics

So what are some of the contributions of ancient Greek mathematicians to the math we use today?

A key Greek contributor, Pythagoras (570 BCE to 495 BCE) is the developer of the Pythagorean Theorem, and made important contributions to religious philosophy and general philosophy as well as mathematics. Indeed, some scholars argue that the ancient Greeks considered mathematics to be a specialized form of natural philosophy and not a separate branch of study at all.

Another ancient Greek philosopher, Thales, used geometry to solve real world problems such as the height of buildings (and the pyramids), and the distance between ships and the shoreline. Plato (428/427 BCE — 348/347 BCE) made important contributions by clarifying the distinction between assumptions and data, as did Euclid (c. 300 BCE), who strengthened the mathematical rigor of proofs by introducing the explicit concepts of definition, axiom, theorem, and proof.

Archimedes (c.287—212 BC) defined the surface area and volume of a sphere and worked with infinite series, as well as contributing to the study of physics with the principle of buoyancy and the creation of the Archimedes screw, which could transfer water from one location to another (including raising it against the pull of gravity).

Chinese and Indian Contributions to Mathematics

Important contributions to ancient mathematics were also made by the Chinese, the Indians, and the Muslims, although the Muslims operated mostly in the eighth century CE and later, so that they are considered to be more modern than ancient times. The Chinese and Indian cultures, however, flourished roughly contemporaneously with the ancient Greeks.

Chinese mathematics, in particular, is so different in its approach and formulations, that scholars generally agree it was developed independently. The Chinese are credited with developing a decimal positional notation system (so that powers of 10, 100, 1000 and so on are distinguished from each other), a variety of geometrical theorems, and mathematical proofs for the Pythagorean Theorem and Gaussian elimination, a technique used in modern day Linear Algebra to manipulate matrices using row reduction.

Unlike ancient Greek math, Chinese mathematics continued to develop well beyond ancient times, so that, for example, in the 13th century CE, Zu Chongzhi calculated the value of pi to seven decimal places and Chu Shih-chieh formulated a method for solving higher order algebraic equations).

The Indian culture for mathematics flourished somewhat later (eighth century BCE to second century CE), and included calculations for the square root of two, a statement (but not proof) of the Pythagorean Theorem, and astronomical treatises from the fourth and fifth centuries CE involving various trigonometric proofs.

Clearly, the mathematics that we use today is the cumulative knowledge of a great many people, stretching back in time more than three thousand years and across a number of different cultures located across the globe.

Exercise 26. Speaking. Answer the following questions

1. When did people begin to count? 2. What purposes did the primitive people use numbers for? 3. Why are mathematics and numbers important? 4. What spheres of our life do we use Maths in? 5. What numeraration system do we use nowadays? 6. How many digits do we use in our Hindu-Arabic system of numeration? 7. Why do we separate figures of the numbers by commas? 8. How is each group of three figures called? 9. How is the system of numbers we use called? 10. How many digits does a period of a number contain? 11. What is the function of a zero?

Individual Work

Task 1. Over to you 1. Web research task. Find out as much as you can about Chaos theory. Web search key words: chaos and the horseshoe, Smale, the butterfly effect, the Lorenz attractor, the Sierpinski gasket, fractals. 2. Prepare a short presentation on the Lorenz attractor. 3. Prepare a short presentation on the applications of nonlinear dynamics

Task 2. Project Ideas Study the literature and analyze your everyday experience to explore the ways of connecting mathematical ideas to real life problems. Discuss your ideas in groups. Choose any field (e.g. economics, science, sports, etc.) of everyday life and research the realworld applications of maths there. Prepare a short report and a presentation on the achieved results. Share your ideas.

Task 3. Follow this link

https://www.ted.com/talks/roger_antonsen_math_is_the_hidden_secret_to_understanding_the_world . Unlock the mysteries and inner workings of the world through one of the most imaginative art forms ever -- mathematics -- with Roger Antonsen, as he explains how a slight change in perspective can reveal patterns, numbers and formulas as the gateways to empathy and understanding. Watch a video about math as the hidden secret to understanding the world as many times as you need. Write out 10 topical theses. Prepare your own presentation on the thema Math as the hidden secret to understanding the world.

Task 4. Follow this link. https://ed.ted.com/best_of_web/U1LBfyZ5 . The Collatz Conjecture is the simplest math problem no one can solve — it is easy enough for almost anyone to understand but notoriously difficult to solve. So what is the Collatz Conjecture and what makes it so difficult? Veritasium investigates. Watch the video as many times as you need. Make sure that you are throughall stages: ”Think”, “Dig Deeper” and “Discuss”. All necessary translations and thoughts you are welcome to take down into your copy-book.

Task 5. Follow this link **https://www.ted.com/talks/samia_talbi_the_real_reason_you_should_study_math .** The benefit and use of math is seriously questioned in many public debates, schools and societies today. Why should we learn mathematical concepts we never use in life, not ever at work? Amid generalizing digitalization, why should children get good at calculation? The present Ted-talk guides us through the mathematical solving process of a tricky problem beleived to be posed by North African queen Dihya. Doing so, it shows that math helps us develop an extremely valuable success skill: conditional creativity. Behind this skill lies our very ability to shape our lives, but also become an attractive person able to make a difference. Watch a video about The Real Reason You Should Study Math as many times as you need. Write out 10 topical theses. Write down the main points and tell about it.

UNIT 2 THE LANGUAGE OF MATHEMATICS

Exercise 1. Read and memorise the following words and word combinations

continuity – безперервність; наступність; нерозривність; цілісність

smoothness – гладкість (напр. функції)

real-valued functions – дійсна функція

functions of complex numbers – функції комплексних чисел

algebraic geometry – алгебраїчна геометрія

number theory – теорія чисел, математична дисципліна, що вивчає властивості чисел

applied mathematics – прикладна математика наукова дисципліна, що вивчає застосування математичних методів в інших галузях знань, у свою чергу поділяється на ряд напрямків

functional analysis - функціональний аналіз

vector spaces - векторний простір

inner product - скалярний твір, внутрішній твір (векторів)

norm - норма вектора (функціонал, заданий на векторному просторі та узагальнюючий поняття довжини вектора або абсолютного значення числа)

topology – топологія

differential and integral equations – диференціальні та інтегральні рівняння

mathematical equation – математичне рівняння

variable – змінна, змінна величина

derivatives of various orders – похідні різного порядку

Exercise 2. Write the transcription and translate the following words. Make sentences with them.

I. ADDITION 3 + 2 = 5

3&2 – ADDENDS

+ – PLUS SIGN

EQUALS SIGN

5 – THE SUM

II. SUBTRACTION – 3 – 2 = 1

3 – THE MINUEND

MINUS

2 – THE SUBTRAHEND

1 – THE DIFFERENCE – різність

III. MULTIPLICATION –3 × 2 = 6

3 –THE MULTIPLICAND

MULTIPLICATION SIGN

2 – THE MULTIPLIER

6 – THE PRODUCT

3&2 –FACTORS

IV. DIVISION 6 : 2 = 3

6 – THE DIVIDEND

DIVISION SIGN

2 – THE DIVISOR

3 – THE QUOTIENT

Exercise 3. Read and translate the text

One of the foremost reasons given for the study of mathematics is to use a common phrase, that – mathematics is the language of science. This is not meant to imply that mathematics is useful only to those who specialized in science. No, it implies that even a layman must know something about the foundations, the scope and the basic role played by mathematics in our scientific age. The language of mathematics consists mostly of signs and symbols, and, in a sense, is an unspoken language.

There can be no more universal or more simple language, it is the same throughout the civilized world, though the people of each country translate it into their own particular spoken language. For instance, the symbol 5 means the same to a person in England, Spain, Italy or any other country; but in each country it may be called by a different spoken word. Some of the best known symbols of mathematics are the numerals 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 and the signs of addition (+), subtraction (-), multiplication (x), division (:), equality (=) and the letters of the alphabets: Greek, Latin, Gothic and Hebrew (rather rarely).

Symbolic language is one of the basic characteristics of modern mathematics for it determines its true aspect. With the aid of symbolism mathematicians can make transition in reasoning almost mechanically by the eye and leave their mind free to grasp the fundamental ideas of the subject matter. Just as music uses symbolism for the representation and communication of sounds, so mathematics expresses quantitatively relations and spatial forms symbolically. Unlike the common language, which is the product of custom, as well as social and political movements, the language of mathematics is carefully, purposefully and often ingeniously designed. By virtue of its compactness, it permits a mathematician to work with ideas which when expressed in terms of common language are unmanageable. This compactness makes for efficiency of thought.

Mathematic language is precise and concise, so that it is often confusing to people unaccustomed to its forms. The symbolism used in math language is essential to distinguish meanings often confused in common speech. Math style aims at brevity and formal perfection. Let us suppose we wish to express in general terms the Pythagorean Theorem, well-familiar to every student through his high-school studies.

We may say: "We have a right triangle. If we construct two squares each having an arm of the triangle as a side and if we construct a square having the hypotenuse of the triangle for its side, then the area of the third square is equal to the sum of the areas of the first two". But no mathematician expresses himself that way. He prefers: "The sum of the squares on the sides of a right triangle equals the square on the hypotenuse." In symbols this may be stated as follows: c2=a2+b2 .

This economy of words makes for conciseness of presentation, and math writing is remarkable because it encompasses much in few words. In the study of mathematics much time must be devoted 1) to the expressing of verbally stated facts in math language, that is, in the signs and symbols of mathematics; 2) to the translating of math expressions into common language. We use signs and symbols for convenience. In some cases the symbols are abbreviations of words, but often they have no such relations to the thing they stand for.

We cannot say why they stand for what they do; they mean what they do by common agreement or by definition. The student must always remember that the understanding of any subject in mathematics presupposes clear and definite knowledge of what precedes. This is the reason why "there is no royal road" to mathematics and why the study of mathematics is discouraging to weak minds, those who are not able to master the subject.

Exercise 4. Which of these statements are true? Correct the false ones.

1. Symbolic language is one of the main characteristics of modern mathematics for it determines its true aspect. 2. The language of mathematics consists of signs and symbols. 3. In the process of studying the mathematics much attention should be devoted: 1- to the expressing of verbally stated facts in math language; 2 - to the translating of math expressions into common language. 4. Like the common language, the language of mathematics is carefully, purposefully and often ingeniously designed. 5. Mathematic language is precise and concise, so that it is often confusing to people unaccustomed to its forms.

Exercise 5. Translate the following sentences into English.

1. У цій системі використовуються позитивні та негативні числа.

2. Позитивні та негативні числа представлені (to represent) відносинами цілих позитивних чисел.

3. Раціональні (rational) числа, своєю чергою, використовуються до створення ірраціональних (irrational) чисел.

4. У сукупності раціональні та ірраціональні числа становлять систему дійсних чисел.

5. Математичний аналіз - це розділ математики, що вивчає функції та межі.

6. Безліч Х є підмножиною іншої множини У тому випадку, якщо всі елементи множини Х одночасно є елементами множини У.

7. Аксіоми, що задовольняють безлічі дійсних чисел, можна умовно поділити на три категорії.

Exercise 6. Translate the following phrases into mathematical expressions or equations

1. Twice a number minus eight 2. The quotient of a number and seven is two 3. Three-fourths of a number. 4. The product of a number and ten is eighty. 5. Eight less than a number is five. 6. How many times does five go into twenty?

Exercise 7. Find derivates in the text for the following words and add the absent forms. Explain their meaning in Ukrainian:

Verb	Noun	Adjective/Adverb
to learn		
		regular
	mystery	
		computational
	Measure	

Exercise 8. Match these terms with their definitions.

1. multiple 2. fraction 3. an irrational number 4. a rational number 5. ratio 6. segment 7. equimultiple	a) any real number that cannot be expressed as the ratio of two integers b) the product of a given number or polynomial and any other one c) a ratio of two expressions or numbers other than zero d) a quotient of two numbers or quantities e) any real number of the form a/b, where a and b are integers and b is not

	zero f) one of the products arising from the multiplication of two or more quantities by the same number or quality g) a part of a line or curve between two points

Exercise 9. Translate from Ukrainian into English.

Поруч із розвитком математики виникли методологічні і філософські праці науку. Прикладом може бути класифікація Гемінуса, грецького астронома і математика 1 в. до н.е. Він вважав, що наука вже нагромадила досить різноманітних відомостей у багатьох областях. Згідно з вивченням Аристотеля, математика вивчає властивості, які можна «абстрагувати» від об'єктів фізичного світу. Крім того, як і всі науки, що ґрунтуються на доказах, вона будується на певних принципах, так що одна наука передбачає існування іншої, одна підкоряється іншій, як казав Аристотель. Приміром, оптика «підпорядковується» геометрії. Що свідчить про існування логічно упорядкованої ієрархії наук. Таку ієрархію слід відрізняти від прийнятого у грецьких вчених протиставлення "практичної" та "чистої" математики. За Аристотелем, тільки остання заслуговує на те, щоб її включили у вільну освіту. "Бути вільним" тут самоціль.

Exercise 10. Match the synonyms.

1. to work out	a) concept
2. to await	b) to take on
3. to apply	c) to use
4. notion	d) to anticipate
5. to hire	e) distinguished
6. to attempt	f) to try
7. noted	g) to develop

8. to break the news	h) scientist
9. to challenge	i) main
10. to research	j) to begin
11. scholar	k) prognosis
12. forecast, n	l) to solve problems connected with
13. to start	m) to investigate
14. major	n) to announce
15. to deal with	o) to put to doubt

Exercise 11. Form nouns from the following verbs by adding suffix "ment". Translate them into Ukrainian.

Example: attach – attachment

1. measure 2. establish 3. equip 4. achieve 5. announce 6. advertise 7. abolish 8. require 9. embarrass 10. punish 11. enchant 12. denounce 13. agree 14. arm 15. disarm 16. accomplish 17. fulfil 18. attach. Exercise 2. Translate the following words into English: 1. озброєння 2. відданість 3. обладнання 4. спростування 5. досягнення 6. скасування 7. вимірювання 10. виконання 11. анонсування 12. вимога 13. покарання 14. домовленість 15. завершення.

Exercise 12. Make up sentences with the following words and wordcombinations 1. abolishment of corporal punishment 2. considerable achievement 3. precise measurement 4. out-of date equipment 5. international agreement 6. modern scientific achievements.

Exercise 13. Match the words on the left with their translation on the right.

1. foundations	a) наука про
2. concise	b) вимір (дія)
3. the study of	c) прикладна
4. measuring	d) сукупність
5. to deal with	e) короткий

6. applied	f) основи
7. pure	g) множини
8. contemporary	h) поняття
9. concept	i) теоретичний
10. mixture	j) розглядати
11. to transform	k) величина
12. to regard	l) кількість
13. to constitute	m) перетворювати
14. magnitude	n) сучасний
15. sets	o) вивчати
16. quantity	p) основи

Exercise 14. Read and translate the text

What is Mathematics? Mathematics is the product of many lands and it belongs to the whole of mankind. We know how necessary it was even for the early people to learn to count and to become familiar with mathematical ideas, processes and facts. In the course of time, counting led to arithmetic and measuring led to geometry. Arithmetic is the study of number, while geometry is the study of shape, size and position.

These two subjects are regarded as the foundations of mathematics. It is impossible to give a concise definition of mathematics as it is a multifield subject. Mathematics in the broad sense of the word is a peculiar form of the general process of human cognition of the real world. It deals with the space forms and quantity relations abstracted from the physical world.

Contemporary mathematics is a mixture of much that is very old and still important (e. g., counting, the Pythagorean theorem) with new concepts such as sets, axiomatics, structure. The totality of all abstract mathematical sciences is called Pure Mathematics. The totality of all concrete interpretations is called Applied Mathematics. Together they constitute Mathematics as a science. One of the modern definitions of mathematics runs as follows: mathematics is the study of relationships among quantities, magnitudes, and properties of logical operations by which unknown quantities, magnitudes and properties may be deduced. In the past, mathematics was regarded as the science of quantity, whether of magnitudes, as in geometry, or of numbers, as in arithmetic, or the generalization of these two fields, as in algebra.

Toward the middle of the 19th century, however, mathematics came to be regarded increasingly as the science of relations, or as the science that draws necessary conclusions. The latter view encompasses mathematical or symbolic logic, the science of using symbols to provide an exact theory of logical deduction and inference based on definitions, axioms, postulates, and rules for combining and transforming positive elements into more complex relations and theorems

Exercise 15. Discussion point What if you were to give a short presentation on Pythagoras's most important contribution to mathematics? What would you say? Do you think Pythagoras formulated and proved the theorem named after him?

Exercise 16. Perfect Tenses. Complete the sentences using the following words: already before ever for just by since so still yet never

1.Have you … dreamt of going to London? 2. I haven't worked out how to set the timer on the video …. 3. My dad's lived in the same house … he was born. 4. The film's only been on … a couple of minutes. 5. Kate has passed three exams out of five … far. 6. He will have translated the text … 3 o'clock tomorrow. 7. He's only … got home. 8. It's eleven o'clock and he … hasn't come home. Where could he be? 9. I've … met Ann …. What's she like? 10. He has … finished doing his homework.

Exercise 17. Transform the sentences from Perfect Active into Perfect Passive.

1. She has just typed her report for the conference. 2. The teacher told us that she had checked all the tests. 3. The student will have written his degree work by May. 4. They have learnt a lot of new English words. 5. He hasn't found the answer yet. 6. I've just received my exam results. 7. By the end of the conference, the participants had discussed a number of important questions concerning the

problem. 8. They will have read two books on topology by the end of the month. 9. We had planned the meeting months in advance, but we still had problems. 10. I had discussed the plan of my work with my science adviser before the end of the class.

Exercise 18. Fill in the gaps using the correct form of the verb in brackets.

All calls …… (register) by the Help Desk staff. Each call …… (evaluate) and then …… (allocate) to the relevant support group. If a visit …… (require), the user…… (contact) by telephone, and an appointment …… (arrange). Most calls …… (deal with) within one working day. In the event of a major problem requiring the removal of a user's PC, a replacement can usually…… (supply).

Exercise 19. Put the verb in brackets into the correct verb form (the Present Simple or the Present Continuous) and then solve the problem.

Imagine you ………… (wait) at the bus stop for a friend to get off a bus from the north. Three buses from the north and four buses from the south ……… (arrive) about the same time. What ………. (be) the probability that your friend will get off the first bus? Will the first bus come / be from the north?

Exercise 20. Complete these sentences by putting the verb in brackets into the Present Simple or the Present Continuous.

a. To solve the problem of gravitation, scientists …………… (consider) time–space geometry in a new way nowadays. b. Quantum rules …………… (obey) in any system. c. We …………… (use) Active Server for this project because it ……… (be) Web–based. d. Scientists …………… (trace and locate) the subtle penetration of quantum effects into a completely classical domain. e. Commonly we ……….. (use) C + + and JavaScript. f. At the moment we …………… (develop) a Web–based project. g. Its domain ………… (begin) in the nucleus and ………… (extend) to the solar system. h. Right now I ………… (try) to learn how to use Active Server properly.

Exercise 21. Read, translate and give a short outline of the text in English

Four Basic Operations of Arithmetic

We cannot live a day without numerals. Numbers and numerals are everywhere. The number names are: zero, one, two, three, four and so on. And here are corresponding numerals: 0, 1, 2, 3, 4. In a numeration system numerals are used to represent numbers. The numbers used in out numeration system are called digits. In our Hindu-Arabic system we use only ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 to represent any number. These digits may be used in various combinations. Thus, for example, 1, 2 and 3 are used to write 123, 213, 132 and so on.

There are four basic operations of arithmetic. They are addition, subtraction, multiplication and division. An equation like $3 + 4 = 7$ represents an operation of addition. An equation is a mathematical sentence that has an equal sign or a sign of equality (=). In this case we say: three plus four is equal to seven. So we add 3 and four and get 7 as a result. 3 and 4 are addends (or summands) and 7 is the sum. There is one more mathematical symbol, a plus sign (+). An equation like $7 - 2 = 5$ represents an operation of subtraction. Here is seven is the minuend and two is a the subtrahend.

As a result of the operation you get 5. It is the difference. We may say that subtraction is the inverse operation of addition since $5 + 2 = 7$ and $7 - 2 = 5$. The same might be said about division and multiplication which are also inverse operations. In multiplication there is a number that must be multiplied. It is the multiplicand.

There is also a multiplier. It is the number by which we multiply. When we are multiplying the multiplicand by the multiplier we get the product as a result. When two or more numbers are multiplied, each of them is called a factor. In the expression five multiplied by two (5Ч2), the 5 and the 2 will be factors.

The multiplicand and the multiplier are names for factors. In the operation of division there is a number that is divided and it is called the dividend; the number by which we divide is called the divisor. When we are dividing the dividend by the divisor we get the quotient. But suppose you are dividing 10 by 3. In this case the divisor will not be contained a whole number of times in the dividend. You will get a part of the dividend left over. This part is called the remainder. In our case the remainder will be 1. Since multiplication and division are inverse operations you may check division by using multiplicaion.

Exercise 22. Solve the following mathematical problems and read them in English:

145 + 478 =

594 – 357 =

510 : 15 =

234 – 167 =

4810 : 65 =

296 +401 =

287 × 56 =

623 × 41 =

742 – 397 =

1260 : 28 =

827 + 511 =

92 × 172 =

Make your own mathematical problems and ask your partners to solve them.

Exercise 23. Speaking. Correct the following sentences:

1. In the Hindu-Arabic numeration system we use five digits. 2. The result of multiplication is called the difference. 3. We get the sum as a result of subtraction. 17 4. Addition and multiplication are inverse operations. 5. Division and subtraction are inverse operations. 6. In the expression 2 + 3 = 5 two and three are factors. 7. In the equation 12 – 11 = 1 one is a minuend. 8. In the mathematical sentence 12 : 6 =2 two is the divisor. 9. If you divide 15 by 4 you will get a whole number as a result. 10. If we multiply 12 by 4 we'll get 50 as a product. 11. The base of the binary system is ten. 12. The decimal system of numeration uses only five digits. 13. The use of parentheses is not important in the following expression 20 × 30 +10 : 2 – 5 + 16. 14. When no parentheses are used in a mathematical sentence it means that first you must add and subtract and then multiply and divide.

15. Any operation is called a binary operation when it is applied to only three numbers at a time.

Exercise 24. Read and translate the text:

Mathematics is associated with numbers and symbols. But, it's also associated with intriguing words. Every mathematical number or symbol has a corresponding word or phrase. Math has a language of its own. The deeper we study the subject, the more likely we will come across words that are unique to this scientific discipline.

Most commonly used words in maths owe their origin to Old English. When the Romans conquered Britain, the local population spoke Celtic. The Romans brought Latin with them and later, other invaders brought their languages. Thus, developed the Anglo-Saxon language aka Old English. Number words such as one, two, three, four and measurement terms such as foot, yard, etc originate from Old English.

French words were introduced after the Norman conquest of England in 1066. Some French origin words include surface and surjection.

Most mathematical terms are of Latin origin. Arithmetic is derived from arithmetica. Interestingly, until the middle ages, most math words had an extra 'r' (eg. arithmetrika).

A large body of mathematical work was produced between 300 BC and 300 AD by Greek mathematicians Euclid and Archimedes among others. They gave us the Greek math words such as isosceles and convex.

Unlike other sciences where new jargon has to be approved by international organisations such as the International Union of Pure and Applied Chemistry (IUPAC), math words formation is left to individual initiative. Therefore the process of word formation in maths is often quite unpredictable. Moreover many specialised words associated with important concepts were not coined for years — mathematicians used the word radius for centuries before 'diameter' came into being. Addition and fraction were coined as early as 1100 AD while determinant came into existence as late as 1810.

Some familiar math words which are part of our daily usage today were heavily opposed when coined. For instance matrix and statistics received a lot of opposition as the origin of these words didn't mean what they denote in mathematics. There is also an abundance of synonyms in this science — e.g modulus and absolute value, characteristic function and indicator function.

Many mathematical concepts such as the Abbe-Helmert Criterion and Zorn's Lemma incorporate the names of people.

Algebra was coined by Persian scholar/astronomer Al-Khowarizmi in 825 AD. The origin Arabic word is al-jabr which means the reunion of broken parts, where a subtracted quantity on one side of an equation became an added quantity when moved to the other side.

Decimal is derived from the Latin decimus, meaning 'tenth'.

Parabola was coined by Greek philosopher Apollonius, who had words for all three conic sections.

Plus or minus were used commonly by the Romans to indicate 'more or less'.

Exercise 25. Answer the questions:

1.Can people live without numerals? 2. How many digits do we use in our HinduArabic system of numeration? 3. Is a base five system used in modern computers? 4. Is subtraction an inverse operation of division? 5. Is addition an inverse operation of multiplication? 6. Are subtraction and addition inverse operations? 7. Are division and multiplication inverse operations? 8. Is the product the result of subtraction? 9. Is the difference the result of division? 10. Will there be a remainder if you divide 36 by 6? 11. Will there be a remainder if you divide 31 by 7? 12. How many basic operations of arithmetic do you know?

Individual Work

Task 1. Over to you 1. Web research task. Find out as much as you can about Galois and his contribution to Group theory. Web search key words: Galois, group theory, quintics, etc.

Task 2. Project. Speak on the Topic “Four Basic Operations of Arithmetic”, give your own examples

Task 3. Follow this link https://www.youtube.com/watch?v=7snnRaC4t5c . Watch the video as many times as you need. Write out 10 topical theses. Prepare your own presentation on the thema Why are people so afraid of mathematics?

Task 4. Follow this link

https://www.youtube.com/watch?v=2RzNYfnBoLA. Every rational number, from the integers, whole numbers, and natural numbers, should be expressible in a fractional form. The infinite non-repeating decimal numbers cannot be expressed as a fraction because of their indefinite value and hence are irration. Watch the video about Types of Numbers | Rational and Irrational as many times as you need. All necessary translations and thoughts you are welcome to take down you’re your copy-book. Write down the main points and tell about it.

Task 5. Follow this link

https://www.ted.com/talks/jeff_dekofsky_is_math_discovered_or_invented.

Would mathematics exist if people didn't? Did we create mathematical concepts to help us understand the world around us, or is math the native language of the universe itself? Jeff Dekofsky traces some famous arguments in this ancient and hotly debated question. Watch a video about the math. Write out 10 topical theses. Write down the main points and tell about it.

UNIT 3. THE FIELDS OF MATHEMATICS

Exercise 1. Read and memorise the following words and word combinations.

real number – дійсне (речове) число

rational number/irrational number – раціональне число/ірраціональне число

integer – ціле число

fraction – дроб, дробове число

square root of – квадратний корінь

π (transcendental number) – трансцендентне число

number line (real line) – цифрова пряма, [речова] цифрова вісь

decimal representation – десяткове уявлення (запис числа в десятковій системі числення)

Pythagoras [piˈthagərəs], [pʌɪˈθagərəs] – Піфагор

negative / positive number – негативне/позитивне число

integral – ціле число

fractional number – дробове число

magnitude – величина; абсолютна величина, значення, модуль

quadratic equations – квадратне рівняння, рівняння другого ступеня рівняння виду Ax2 + Bx + C = 0, де A не дорівнює нулю

coefficient – коефіцієнт; множник

equation – рівняння; рівність

cube root – кубічний корінь, корінь третього ступеня

fourth root – корінь четвертого ступеня

decimal notation – десяткова система обчислення

Exercise 2. Write the transcription and translate the following words. Make sentences with them.

fraction line

improper fraction

integer c

numerator

lowest terms

G. C. D.

proper fraction

value

division

common fraction

rational number

mixed number

reducing a fraction m.

quantity

stand for

equal parts

Exercise 3. Read and translate the text

Mathematics can be subdivided into the study of structure, quantity, space, and change. There are also subdivisions dedicated to exploring links from mathematics to other fields: to logic, to set theory (foundations), to the empirical mathematics of the various sciences (applied mathematics), and more recently to the rigorous study of uncertainty.

The study of quantity begins with numbers, first the familiar natural numbers and integers ("whole numbers") and arithmetical operations on them, which are characterized in arithmetic. The deeper properties of integers are studied in number theory, from which come such popular results as Fermat's Last Theorem. As the number system is further developed, the integers are recognized as a subset of the rational numbers ("fractions").

These, in turn, are contained within the real numbers, which are used to represent continuous quantities. Real numbers are generalized to complex numbers. Discussion of the natural numbers leads to the transfinite numbers, which formalize the concept of "infinity".

Another area of study is size, which leads to the cardinal numbers and then to another conception of infinity: the aleph numbers, which allow meaningful comparison of the size of infinitely large sets.

Many mathematical objects, such as sets of numbers and functions, exhibit internal structure. The structural properties of these objects are investigated in the study of groups, rings, fields and other abstract systems, which are themselves such objects. This is the field of abstract algebra.

An important concept here is that of vectors, generalized to vector spaces, and studied in linear algebra. The study of vectors combines three of the fundamental areas of mathematics: quantity, structure, and space. A number of ancient problems concerning Compass and straightedge constructions were finally solved using Galois theory.

The study of space originates with geometry – in particular, Euclidean geometry. Trigonometry is the branch of mathematics that deals with relationships between the sides and the angles of triangles and with the trigonometric functions; it combines space and numbers, and encompasses the well-known Pythagorean theorem.

The modern study of space summarizes these ideas to include higherdimensional geometry, non-Euclidean geometries and topology. Quantity and space both play a role in analytic geometry, differential geometry, and algebraic geometry. Within differential geometry are the concepts of fiber bundles and calculus on manifolds, in particular, vector and tensor calculus. Within algebraic geometry is the description of geometric objects as solution sets of polynomial equations, combining the concepts of quantity and space, and also the study of topological groups, which combine structure and space. Lie groups are used to study space, structure, and change.

Topology in all its many ramifications may have been the greatest growth area in 20th century mathematics; it includes point-set topology, set-theoretic topology, algebraic topology and differential topology. In particular, instances of modern day topology are metrizability theory, axiomatic set theory, homotopy theory, and Morse theory.

Topology also includes the now solved Poincaré conjecture and the controversial four color theorem, whose only proof, by computer, has never been verified by a human. To understand and describe change is a common theme in the natural sciences, and calculus was developed as a powerful tool to investigate it. Functions arise here, as a central concept describing a changing quantity.

The rigorous study of real numbers and functions of a real variable is known as real analysis, with complex analysis the equivalent field for the complex numbers. Functional analysis focuses attention on (typically infinite-dimensional) spaces of functions. One of many applications of functional analysis is quantum mechanics. Many problems lead naturally to relationships between a quantity and its rate of change, and these are studied as differential equations. Many phenomena in nature can be described by dynamical systems; chaos theory makes precise the ways in which many of these systems exhibit unpredictable yet still deterministic behavior.

Exercise 4. Give the English equivalents of the following Ukrainian words and word combinations:

Відношення цілих, множник, абсолютний квадрат, аксіома порядку, розкладання на множники, рівняння, приватне, раціональне число, елементарні властивості, певне раціональне число, квадратний, протиріччя, доказ, середнє (значення).

Exercise 5. Translate from Ukrainian into English.

Математика – це наука про числа та кількості, про структури, порядки та відносини, що до неї входять арифметика та алгебра, геометрія та тригонометрія, тощо. Математика на відміну природничих наук, вивчає не явища природи, а логічні побудови, тому експерименти в математиці не випробуванням природи, а випробуванням гіпотез за умов логіки. Рахунок став початком математики. Люди з математикою стикалися з давніх-давен.

Наприклад, щоб визначити, хто багатший? Чи у кого більше худоби? Родоначальниками математики визнано греки (6–4 ст. до н.е.). Математика ділилася на арифметику та логістику. У Середні віки (близько 400-1100) рівень математичного знання не піднімався вище за арифметику, але важливим розділом математики в той період вважалася астрологія. У Західній Європі в 16 столітті були введені в обіг десяткові дроби та правила арифметичних дій з ними. На початку 19 століття математиків продовжувало займати основне завдання алгебри – пошук загального розв'язання рівнянь алгебри. Жоден математик сьогодні може сподіватися знати більше, що відбувається у дуже маленькому куточку науки.

Exercise 6. Complete the table.

Weak describe compete persuasive appreciation marriage react strong suspect education apologize sympathetic deep conclude warm sweetness prefer explosion discouraging short close attend discovered power reliability special

NOUN	VERB	ADJECTIVE

Exercise 7. Match the terms from the left column and the definitions from the right column:

1.perfect square 2.factor 3. multiple 4. average 5. factorization	a) any of two or more quantities which form a product when multiplied together b) the numerical result obtained by dividing the sum of two or more

	quantities by the number of quantities c) the process of finding the factors d) a number which is a product of some specified number and another number e) a quantity which is the exact square of another quantity

Exercise 8. Translate the mathematical expression into English

1. $x + 12 = 8$ 2. $3x = 15$ 3. $x/15$ 4. $10/x$ 5. $x – 6$

Exercise 9. Form nouns from the following verbs by adding suffix -“ion” (t “ion” or –s “ion”). Translate them into Ukrainian.

Example: transport – transportation; transmit – transmission.

1. constitute 2. add 3. subtract 4. translate 5. reduce 6. repeat 7. compete 8. pollute 9. exhibit 10. divide 11. collide 12. intrude 13. compare 14. assume 15. decide 16. prolong 17. deduce 18. induce 19. implement.

Exercise 10. Form nouns from the following verbs:

Example: multiply – multiplication

1. classify 2. imply 3. amplify 4. codify 5. purify 6. mistify 7. glorify 8. justify 9. qualify 10. personify 11. verify 12. rectify 13. specify 14. certify 15. simplify.

Exercise 11. Read and translate the text

Mathematics and Art

Mathematics and its creations belong to art rather than science. It is convenient to keep the old classification of mathematics as one of the sciences, but it is more appropriate to call it an art or a game. Unlike the sciences, but like the art of music or a game of chess, mathematics is foremost a free creation of the human mind. Mathematics is the sister, as well as the servant of the arts and is touched with the same genius. In the age when specialization means isolation, a layman may be surprised to hear that mathematics and art are intimately related. Yet, they are closely identified from ancient times.

To begin with, the visual arts are spatial by definition. It is, therefore, not surprising that geometry is evident in classic architecture or that the ruler and compass are as familiar to the artist as to the artisan. Artists search for ideal proportions and mathematical principles of composition. Many trends and traditions in this search are mixed. Mathematics and art are mutually indebted in the area of perspective and symmetry which express relations only now fully explained by the mathematical theory of groups, a development of the last centuries.

Exercise 12. Make a short report on the Topic "Who invented fraction?"

Exercise 13. Ask special questions using question words given in parentheses.

The development of geometry 1. The earliest recorded beginnings of geometry can be traced to early predecessors. (to whom) 2. They discovered obtuse triangles in the ancient Indus Valley and ancient Babylonia from around 3000 BC. (where; when) 3. Early geometry was a collection of empirically discovered principles concerning lengths, angles, areas, and volumes. (what collection) 4. In geometry a spatial point is a primitive notion upon which other concepts may be defined. (where) 5. Points have neither volume, area, length, nor any other higher dimensional analogue. (what (question to the subject)) 6. In branches of mathematics dealing with a set theory, an element is often referred to as a point. (where; how) 7. A point could also be defined as a sphere which has a diameter of zero. (how)

Exercise 14. Translate the following words into English:

1. містифікація 2. виправдання 3. очищення 4. розширення 5. втілення 6. підтвердження 7. зіткнення 8. припущення 9. повторення 10. забруднення 11. рішення 12. продовження 13. спрощення 14. виправлення 15. вторгнення.

Exercise 15. Discuss the following questions:

1. The development of mathematics in early civilizations. 2. The contribution of Greek mathematicians. 3. Major progress in mathematics during the epoch of

Renaissance. 4. The development of mathematics in the 17th and 18th centuries. 5. Unsolved problems in mathematics. 6. Modern trends in mathematics.

Exercise 16. Put "can", "can not", "could", "could not" into the following sentences.

a. Parents are finding that they ………….. no longer help their children with their arithmetic homework. b. The solution for the construction problems …………… be found by pure reason. c. The Greeks …………….. solve the problem not because they were not clever enough, but because the problem is insoluble under the specified conditions. d. Using only a straight-edge and a compass the Greeks …………. easily divide any line segment into any number of equal parts. e. Web pages…………. offer access to a world of information about and exchange with other cultures and communities and experts in every field.

Exercise 17. The present simple or the past simple. Put the verbs in brackets in the correct forms.

a. The problem of constructing a regular polygon of nine sides which …………..(require) the trisection of a 600 angle ……… (be) the second source of the famous problem. b. The Greeks ……… (add) "the trisection problem" to their three famous unsolved problems. It ……… (be) customary to emphasize the futile search of the Greeks for the solution. c. The widespread availability of computers …………… (have) in all, probability changed the world for ever. d. The microchip technology which ………… (make) the PC possible has put chips not only into computers, but also into washing machines and cars. e. Fermat almost certainly ………… (write) the marginal note around 1630, when he first ………… (study) Diophantus's Arithmetica. f. I ………… (protest) against the use of infinitive magnitude as something completed, which ……… (be) never permissible in maths, one ………… (have) in mind limits which certain ratio ……….. (approach) as closely as desirable while other ratios may increase indefinitely (Gauss). g. In 1676 Robert Hooke ……………(announce) his discovery concerning springs. He ……………..(discover) that when a spring is stretched by an increasing force, the stretch varies directly according to the force.

Exercise 18. Put the verbs in the brackets into the Present Perfect tense and read through this extract from an advertisement about the Emerging Markets Fund.

Over the past five years, the capital returns from many emerging Asian and Latin American stock markets …………… (be) substantially higher than those of the developed world. For example the market in Argentina ………………(rise) by 793% and Mexico (increase) by 645%. In Asia, the booming market in Thailand ………... (go up) by 364%, and investors in the Philippines ……………… (see) a return of 204%. The major developed nations …………… (not / manage) to make anything like such significant returns. The market in the USA ………… (grow) by 69.8% and in Japan, the market (fall) ………………… by 32.2% over the same period. The growth rates that these emerging markets …………….. (enjoy) in recent years is little short of phenomenal. And we are firmly convinced, much more is yet to come. Our new Emerging Markets Fund, therefore, offers you an easy and attractive way of investing now in the world of tomorrow and its many exceptional growth opportunities.

Exercise 19. Translate the following sentences into English.

1. У останьому випадку обидві теореми – як пряма, так і зворотня – виявляються справедливими. 2. П'ять аксіом Евкліда – це пропозиції, що вводять відносини рівності чи нерівності величин. 3. Підручник Евкліда з геометрії «Початки» читали, читають і читатимуть багато людей. 4. Пропозиція, яка випливає безпосередньо з аксіоми, називається наслідком. 5. Наступні дві теореми обернені один до одного. 6. Одна й та сама пропозиція може бути або не бути істинною щодо другої множини припущень. 7. У будь-якій теоремі є дві частини: гіпотеза та висновок. 8. Вас просять записати коротко припущення, що ви зробили. 9. Аксіома – це справжнє, вихідне положення теорії. 10. Постулат – це твердження, яке у будь-якій науковій теорії як істинне, хоч і доведене її засобами, і тому він відіграє у ній роль аксіоми.

Exercise 20. Read and translate the text.

The main branches of mathematics are algebra, number theory, geometry and arithmetic. Based on these branches, other branches have been discovered. Before the advent of the modern age, the study of mathematics was very limited. But over a period of time, mathematics has been developed as a vast and diverse topic. Development in Maths continues making large contributions to the field of technology. Hence, it is better termed as the Queen of Science.

From the early number system to the modern research areas of computational sciences and probability, a number of new areas have evolved with the base of mathematics. With the expansion of the scope and usage of the subject, there is a corresponding need to classify different branches of mathematics.

What are the Branches of Mathematics?

Mathematics can be broadly grouped into the following branches:

- **Arithmetic**: It is the oldest and the most elementary among other branches of mathematics. It deals with numbers and the basic operations- addition, subtraction, multiplication, and division, between them.
- **Algebra**: It is a kind of arithmetic where we use unknown quantities along with numbers. These unknown quantities are represented by letters of the English alphabet such as X, Y, A, B, etc. or symbols. The use of letters helps us to generalize the formulas and rules and also helps you find the unknown missing values in the algebraic expressions and equations.
- **Geometry**: It is the most practical branch of mathematics that deals with shapes and sizes of figures and their properties. The basic elements of geometry are points, lines, angles, surfaces and solids.

There are some other branches of mathematics that you would deal with in the higher classes.

- **Trigonometry**: Derived from two Greek terms, i.e., trigon (means a triangle) and metron (means measurement), it is the study of relationships between angles and sides of triangles.

- **Analysis**: It is the branch that deals with the study of the rate of change in different quantities. Calculus forms the base of analysis.

Exercise 21. Speaking. Answer the following questions:

1.What does a fraction represent? 2. What do we call “the terms of fractions”? 3. What is the numerator? 4. What is the denominator? 5. What does a fraction indicate? 6. When is the fraction equal to 1? 7. What is a common fraction called? Give the examples. 8. What is a proper fraction called? Give your examples. 9. Is the value of a proper fraction more or less than 1? 10. What do we call mixed numbers? Give your examples. 11. How do you reduce a fraction to its lower terms? 12. What is an improper fraction called? Give your examples.

Individual Work

Task 1. Make up True / False statements about different types of fractions. Ask your group mates.

Task 2. Over to you 1. Web research task. Find out as much as you can about Topology and its applications. Web search key words: hole, knot, vertex, Riemann surface, Klein bottle, Poincare, Perelman.

Project. Make a short report on the Topic "Who invented fraction?"

Task 3. Follow this link. Watch video as many times as you need. All necessary translations and thoughts you are welcome to take down you're your copy-book.

https://www.ted.com/talks/eddie_woo_how_math_is_our_real_sixth_sense. Write down the main points and tell about it.

Task 4. Follow this link https://ed.ted.com/lessons/can-you-solve-the-time-travel-riddle-dan-finkel. Watch the video as many times as you need. Make sure that you are throughall stages: "Think", "Dig Deeper" and "Discuss". All necessary translations and thoughts you are welcome to take down into your copy-book.

Task 5. Follow this link https://ed.ted.com/lessons/how-folding-paper-can-get-you-to-the-moon?lesson_collection=math-in-real-life. Watch the video How folding paper can get you to the moon as many times as you need. Make sure that you are throughall stages: "Think", "Dig Deeper" and "Discuss". All necessary translations and thoughts you are welcome to take down into your copy-book.

UNIT 4. WHAT MATHEMATICS STUDIES

Exercise 1. Read and memorise the following words and word combinations.

Addition – додавання

multiplication – множення

ordered field – впорядковане поле

total order (linear order, total order, simple order, non-strict ordering) – лінійно впорядковане безліч або ланцюг

non-empty subset – непусте підмножина

upper bound – верхня межа, верхня межа

least upper bound – точна (найменша) верхня грань (кордон), або супремум

converge – 1) сходитися; прагнути до (загальної) межі 2) зводити (в-одну точку)

construction of the real numbers – конструктивні способи визначення речовинного числа

limit – ліміт, межа

exponential function – експонентна функція, показова функція

lattice-complete – повна решітка, частково впорядкована множина, в якій всяке непусте підмножина A має точну верхню і нижню грань, звані зазвичай об'єднанням та перетином елементів підмножини A.

Exercise 2. Write the transcription and translate the following words. Make sentences with them.

a mixed number

like fractions

reduction

equivalent fractions

multiplication

unlike fractions 7

a remainder

a quotient

simple order

Exercise 3. Read and translate the text

History teaches the continuity of the development of science. We know that every age has its own problems, which the following age either solves or casts aside as profitless and replaces by new ones. If we would obtain an idea of the probable development of mathematical knowledge in the immediate future, we must let the unsettled questions pass before our minds and look over the problems which the science of today sets and whose solution we expect from the future. To such a review of problems the present day, lying at the meeting of the centuries seems to be well adapted. For the close of a great epoch not only invites us to look back into the past but also directs our thoughts to the unknown future. The deep significance of certain problems for the advance of mathematical science in general and the important role which they play in the work of the individual investigator are not to be denied.

As long as a branch of science offers an abundance of problems, so long is it alive; a lack of problems foreshadows extinction or the cessation of independent development. Just as every human undertaking pursues certain objects, so also mathematical research requires its problems. It is by the solution of problems that the investigator tests the temper of his steel; he finds new methods and new outlooks, and gains a wider and freer horizon. It is difficult and often impossible to judge the value of a problem correctly in advance; for the final award depends upon the gain which science obtains from the problem.

Are there general criteria which mark a good mathematical problem? An old French mathematician said: "A mathematical theory is not to be considered complete until you have made it so clear that you can explain it to the first man whom you meet on the street."

This clearness and ease of comprehension, here insisted on for a mathematical theory, I should still more demand for a mathematical problem if it is to be perfect; for what is clear and easily comprehended attracts, the complicated repels us. Moreover a mathematical problem should be difficult in order to entice us, yet not completely inaccessible, lest it mock at our efforts. It should be to us a guide post on the mazy paths to hidden truths, and ultimately a reminder of our pleasure in the successful solution.

The mathematicians of past centuries were accustomed to devote themselves to the solution of difficult particular problems with passionate zeal. They knew the value of difficult problems. For example, the "problem of the line of quickest descent," proposed by John Bernoulli.

Experience teaches, explains Bernoulli, that lofty minds are led to strive for the advance of science by nothing more than by laying before them difficult and at the same time useful problems, and he therefore hopes to earn the thanks of the mathematical world by following the example of men like Mersenne, Pascal, Fermat, Viviani and others and laying before the distinguished analysts of his time a problem by which, as a touchstone, they may test the value of their methods and measure their strength. The calculus of variations owes its origin to this problem of Bernoulli and to similar problems.

Exercise 4. Put the statements in the right order.

1. Mathematics helps to find solutions to various problems. 2. The solution of the problem depends on the perspective of privilege that will get the science. 3. There are different problems in each time of our life. 4. Mathematical problem should be difficult in order to entice us. 5. Lack of problems foreshadows extinction or the cessation of independent development.

Exercise 5. Which of these statements are true? Correct the false ones.

1. The mathematicians of past centuries were not accustomed to devote themselves to the solution of difficult particular problems. 2. The deep significance of certain problems for the advance of mathematical science in general and the important role which they play in the work of the individual investigator are

denied. 3. It is difficult and often impossible to judge the value of a problem correctly in advance. 4. A mathematical problem should be simple in order to entice us. 5. To solve a mathematical problem meant to find its complete numerical solution. 6. Existence and uniqueness of the problem should be required if it is to be called correctly posed. 7. The mathematical problem is the appropriate expression of an actual physical situation. 8. If the solution depends continuously on the data, it may be called unstable.

Exercise 6. Translate the following sentences into English and answer to the questions in pairs.

1. Які числа називаються раціональними?
2. Які аксіоми використовуються для багатьох раціональних чисел?
3. Скільки раціональних чисел може бути між двома будь-якими раціональними числами?
4. Дійсні числа, що не є раціональними, належать до категорії ірраціональних чисел, чи не так?

Exercise 7. Translate the text from Ukrainian into English.

Зазвичай нелегко довести, що є ірраціональним. Не існує, наприклад, простого доказу ірра-ційності числа eπ. Проте, неважко встановити ірраціональність певних чисел, таких як √2, і фактично можна легко довести

наступну теорему: якщо n є позитивним цілим числом, яке не відноситься до абсолютних квадратів, √n є ірраціональним.

Exercise 8. Translate the mathematical expression into English

6. $5(x + 4)$ 7. $2(x-3) =12$ 8. $7/x$ 9. $(6 –x)/9$ 10. $4(12+y)$

Exercise 9. Match the following terms with definitions and translate them into Ukrainian.

a. algebra	1. a physical quantity having magnitude and direction, represented by a directed arrow indicating its orientation in space
b. triangle	2. a step by step procedure by which an operation can be

	carried out
c. arithmetic	3. a proposition that is not actually proved or demonstrated, but is considered to be self-evident and universally accepted as a starting point for deducing and inferring other truths and theorems, without any need of proof
d.coordinate	4. the ordered pair that gives the location or position of a point on a coordinate plane, determined by the point's distance from the x and y axes
e. axiom	5. the part of mathematics that studies quantity, especially as the result of combining numbers (as opposed to variables) using the traditional operations of addition, subtraction, multiplication and division
f. algorithm	6. a polygon with three edges and three vertices, e.g. a triangle with vertices A, B, and C is denoted $\Delta\ ABC$
g. vector	7. a branch of mathematics that uses symbols or letters to represent variables, values or numbers, which can then be used to express operations and relationships and to solve equations

Exercise 10. Match the terms from the left column and the definitions from the right column:

an axis	a prescribed collection of points, numbers or other objects satisfying the given condition
a scale	the act or result of interpretation; explanation, meaning
an axiom	a straight line through the center of a plane figure of a solid, especially one around which the parts are symmetrically arranged

complex	a system of numerical notation
a point	not simple, involved or complicated
an inequality	a statement or proposition which needs no proof because its truth is obvious, or one hat is accepted as truee without proof
a set	the relation between two unequal quantities, or the expression of this relationship
interpretation	an element in geometry having definite position, but no size, shape or extension

Exercise 11. Form adjectives from the following nouns by adding suffix - "ful". Translate them into Ukrainian.

Example: care – careful

1. hope 2. thought 3. tact 4. meaning 5. play 6. fruit 7. pain 8. colour 9. help 10. use 11. duty 12. harm.

Exercise 12. Form adjectives from the following nouns by adding suffix - "less". Translate them into Ukrainian.

2. Example: care – careless

1. home 2. fruit 3. tact 4. meaning 5. list 6. mother 7. penny 8. hope 9. sense 10. colour 11. help 12. harm.

Exercise 13. Give the English equivalents of the following words and word combinations:

відповідний кут, тупокутний трикутник, дотична дуга, хорда, кільце, коло, простір, рівняння прямої у відрізках, вектор положення точки, просторова крива, прямолінійна координата, за годинниковою стрілкою, проти годинникової стрілки, кут обертання, опуклий багатокутник рівнокутний багатокутник.

Exercise 14. Read and translate the text

Algebra

The earliest records of advanced, organized mathematics date back to the ancient Mesopotamian country of Babylonia and to the Egypt of the 3rd millennium BC. Ancient mathematics was dominated by arithmetic, with an emphasis on measurement and calculation in geometry and with no trace of later mathematical concepts such as axioms or proofs.

It was in ancient Egypt and Babylon that the history of algebra began. Egyptian and Babylonian mathematicians learned to solve linear and quadratic equations as well as indeterminate equations whereby several unknowns are involved.

The Alexandrian mathematicians Hero of Alexandria and Diophantus continued the traditions of Egypt and Babylon, but Diophantus' book Arithmetica is on a much higher level and gives many surprising solutions to difficult indeterminate equations. In the 9th century, the Arab mathematician Al-Khwarizmi wrote one of the first Arabic algebras, and at the end of the same century, the Egyptian mathematician Abu Kamil stated and proved the basic laws and identities of algebra.

By medieval times, Islamic mathematicians had worked out the basic algebra of polynomials; the astronomer and poet Omar Khayyam showed how to express roots of cubic equations. An important development in algebra in the 16th century was the introduction of symbols for the unknown and for algebraic powers and operations. As a result of this development, Book 3 of La geometria (1637) written by the French philosopher and mathematician Rene Descartes looks much like a modern algebra text. Descartes' most significant contribution to mathematics, however, was his discovery of analytic geometry, which reduces the solution of geometric problems to the solution of algebraic ones

Exercise 15. True or false?

1. In the 3rd millennium BC, mathematics was dominated by arithmetic. 2. The history of algebra began in Europe. 3. The book Arithmetica was written by

Diophantus. 4. One of the first Arabic algebras was written by the Arab mathematician AlKhwarizmi. 5. The basic algebra of polynomials was worked out by Rene Descartes. 6. Omar Khayyam introduced symbols for the unknown and for algebraic powers and operations. 7. Analytic geometry was discovered by Islamic mathematicians.

Exercise 16. Answer the following questions.

1. What was characteristic of ancient Mathematics? 2. Where did the history of algebra begin? 3. What equations did Egyptian and Babylonian mathematicians learn to solve? 4. Who continued the traditions of Egypt and Babylon? 5. Who was algebra developed by in the 9th century? 6. What mathematicians advanced algebra in medieval times? 7. What was an important development in algebra in the 16th century? 8. What was the result of this development? 9. What was Rene Descartes' most significant contribution to mathematics?

Exercise 17. Match the words

1. contribution	a) рішення
2. development	b) вклад
3. solution	c) досягнення
4. records	d) ступінь
5. quadratic	e) кубічний
6. to work out	f) розробляти
7. polynomial	g) відкриття
8. unknown	h) невідоме
9. discovery	i) багаточлен
10. ancient	j) корінь
11.indeterminate	k) стародавній
12.identity	l) невизначений
13.root	m) тотожність
14.power	n) письмові матеріали
15.cubic	o) квадратний

Exercise 18. Translate the text into English.

Призмою називається багатогранник, утворений ув'язненими між двома паралельними площинами відрізками всіх паралельних прямих, які перетинають плоский багатокутник в одній із площин. Грані призми, що у цих площинах, називаються підставами призми. Інші грані називаються бічними гранями. Усі бічні грані – паралелограми. Ребра призми, що з'єднують вершини основ, називаються бічними ребрами. Усі бічні ребра призми паралельні.

Висотою призми називається відстань між площинами її основ. Відрізок, що з'єднує дві вершини, що не належать до однієї грані, називається діагоналлю призми. Призма називається прямою, якщо її бічні ребра перпендикулярні до основ. Інакше призма називається похилою. Пряма призма називається правильною, якщо її основи є правильними багатокутниками.

Exercise 19. Give the best English equivalents for the words in parentheses.

1. A circle is (найпростіша) of all curved lines. 2. Every point at a distance (більше) than radius (кажуть) to be outside the circle. 3. A secant segment is a line segment with an endpoint in the exterior of a circle, and the other endpoint on the circle, (найдальнішою) from the external point. 4. Tom comes top in all the exams – he must be (найрозумніший) student in the group. 5. (Чим менше) students think, (тим більше) they talk. 6. How are you today? – I'm very (добре), thanks. 7. Is this proof (більш правильно)? 8. Peter speaks English (найшвидше) of all the students in this group. 9. (Чим більше) I learn, (тим більше) I forget and (тим менше) I know. 10. (Чим скоріше) the problem is solved, (тим краще). 11. This contribution of the ancient Greeks is (набагато більше, ніж) the formulas of the Egyptians.

Exercise 20. Write the comparative and superlative of the words below.

new tiny common bad soon shallow gentle little convenient clever badly many easily complex good much

Exercise 21. Write the words in brackets in the correct form of the degrees of comparison.

a. We all use this method of research because it is ………………. (interesting) the one we followed. b. I could solve quicker than he because the equation given to me was……….(easy) the one he was given. c. The remainder in this operation of division is …………….. (great) than 1. d. The name of Leibnitz is …………….. (familiar) to us as that of Newton. e. Laptops are ……………………. (powerful) microcomputers. We can choose either of them. f. A mainframe is……… (large) and…………..(expensive) a microcomputer. g. One of the ………… (important) reasons why computers are used so widely today is that almost every big problem can be solved by solving a number of little problems. h. Even the …………(sophisticated) computer, no matter how good it is, must be told what to do.

Exercise 22. Put the words in brackets into the correct form to make an accurate description of sizes of computers.

There are different types of computer. The (large)1 …………… and (powerful)2 ……………… are mainframe computers. Minicomputers are (small)3 ……… … than mainframes but are still very powerful. Microcomputers are small enough to sit on a desk. They are the (common)4 ……………. type of computer. They are usually (powerful)5 …………….. than minicomputers. Portable computers are (small)6 ………….. than desktops. The (large)7 ……… portable is a laptop. (Small)8 …………… portables, about the size of a piece of writing paper, are called notebook computers. Subnotebooks are (small)9 ………….. than notebooks. You can hold the (small)10 …………… computers in one hand. They are called handheld computers or palmtop computers.

Exercise 23. Complete sentences using should, must or have to with the verb in brackets.

a. It has been required that he ……………(read) his paper at the seminar. b. After finding the solution, we ………………(say) that axiom and its properties are important enough. c. Scientists ………………(develop) this branch of mathematics, I think. d. She ………..(summarize) the result before she reports it to her boss. e. You ………………(distinguish) between maths objects e.g. numbers, sets of numbers, functions, mappings, transformations, etc. f. The two rays of an angle …………..(not lie) on the same straight line. g. I think you ...……………(illustrate) this problem in the figure. This may be the easiest way. h. In geometry, set notation and language …………… (clarify) matters. i. A polygon …………… (not have) less than 3 segments.

Exercise 24. Read and translate the text

In keeping with modern semiotics I want to understand a text as a simple or compound sign that can be represented as a selection or combination of spoken words, gestures, objects, inscriptions using paper, chalkboards or computer displays, as well as recorded or moving images. Mathematical texts can vary from, at one extreme in research mathematics, printed documents that utilize a very restricted and formalized symbolic code, to the other extreme, multimedia and multi modal texts, such as those used in kindergarten arithmetic. These can include a selection of verbal sounds and spoken words, repetitive bodily movements, arrays of sweets, pebbles, counters, and other objects, including specially designed structural apparatus, sets of marks, icons, pictures, written language numerals and other writing, symbolic numerals, and so on. The received view is that progression in the teaching and learning of mathematics involves a shift in texts from the informal multi-modal to the restrictive, rigorous symbol-rich written text. It is true that, for some, access to the heavily abstracted and coded texts of mathematics grows through the years of education from kindergarten through primary school, secondary school, high school, college, culminating in graduate studies and research mathematics. But it is a myth that informal and multi-modal texts disappear in higher level mathematics.

What happens is that they disappear from the public face of mathematics, whether these be in the form of answers and permitted displays of ‘workings’, or calculations in work handed in to the school mathematics teacher, or the standard accepted answer styles for examinations, or written mathematics papers for publication. As Hersh (1988) has pointed out, mathematics (like the restaurant or theatre) has a front and a back.1 What is displayed in the front for public viewing is tidied up according to strict norms of acceptability, whereas the back (where the preparatory work is done) is often messy and chaotic.

The difference between displayed mathematical texts, at all levels, and private ‘workings’ underscores the rhetorical norms that tidy texts into modes of public address. These norms concern how mathematical texts must be written, styled, structured and presented in order to serve a social function, namely to persuade the intended audience that they represent the knowledge of the writer. Rhetorical norms are social conventions that serve a gatekeeper function.

They work as a filter imposed by persons or institutions that have power over the acceptance of texts as mathematical knowledge representations. Rhetorical norms and standards are applied locally, and they usually include idiosyncratic local elements, such as how a particular teacher or an examinations board likes answers laid out, and how a particular journal requires references to other works to be incorporated. Thus, one inescapable feature of the mathematical text is its style, reflecting its purpose and most notably, its rhetorical function. Rhetoric is the science or study of persuasion, and its universal presence in mathematical text serves to underscore the fact that mathematical signs or texts always have a human or social context.

Exercise 25. Discussion point Your Mathematics and Mechanics Faculty is trying to choose a new course for the coming year. The dean’s office narrowed the list of suggestions down to two possibilities. You are part of the student committee that has been asked to recommend one of the courses. Course 1. The P=NP problem. Course 2. The RSA cryptosystem. Discuss these courses in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Individual Work

Task 1. Over to you 1. Speak about the origin of Number Theory. Comment on the role of Number theory in Internet commerce. 2. Web research tasks. Research applications of number theory and present your findings to the class. The template in Unit 3 may help. Web search key words: cryptography, internet commerce, RSA code, security codes, decoding messages 3. Work in two teams. You are members of a conference committee. You are going to organize a conference on the topic “Number theory”. As a group make a list of research problems to be discussed within different workshops or an information bulletin containing a brief summary of all the workshop discussion points to attract prospective participants.

Task 2. Project. Speak on the Topic “Rational Numbers”, give your own examples.

Task 3. Follow this link https://ed.ted.com/lessons/one-is-one-or-is-it?lesson_collection=math-in-real-life . One bag of apples, one apple, one slice of apple -- which of these is one unit? Explore the basic unit of math (explained by a trip to the grocery store!) and discover the many meanings of one. Watch the video One is one...or is it? as many times as you need. Make sure that you are throughall stages: ”Think”, “Dig Deeper” and “Discuss”. All necessary translations and thoughts you are welcome to take down into your copy-book.

Task 4. Follow this link. [https://www.ted.com/talks/wajdi_mohamed_ratemi_the_mathematical_secrets_of_pascal_s_triangle.](https://www.ted.com/talks/wajdi_mohamed_ratemi_the_mathematical_secrets_of_pascal_s_triangle) Pascal's triangle, which at first may just look like a neatly arranged stack of numbers, is actually a mathematical treasure trove. But what about it has so intrigued mathematicians the world over? Wajdi Mohamed Ratemi shows how Pascal's triangle is full of patterns and secrets. Watch the video as many times as you need. Write out 10 topical theses. Prepare your own presentation on the thema The mathematical secrets of Pascal's triangle.

Task 5. Follow this link. https://www.ted.com/talks/jim_simons_the_mathematician_who_cracked_wall_street. Jim Simons was a mathematician and cryptographer who realized: the complex math he used to break codes could help explain patterns in the world of finance. Billions later, he's working to support the next generation of math teachers and scholars. TED's Chris Anderson sits down with Simons to talk about his extraordinary life in numbers. Watch the video about The mathematician who cracked Wall Street as many times as you need. Write out 10 topical theses. Write down the main points and tell about it.

UNIT 5. DECIMAL FRACTIONS

Exercise 1. Read and memorise the following words and word combinations

Fractional component – дробова складова

set of integers – безліч цілих чисел

natural numbers – натуральні числа

additive inverse – адитивна інверсія, інверсія щодо складання

boldface – напівжирний шрифт, напівжирний (про шрифт)

blackboard bold – спосіб написання жирним шрифтом

countably infinite – рахунково нескінченний

algebraic number theory – алгебраїчга теорія чисел

algebraic integers – ціле число алгебри

number line – [речова] цифрова вісь

exponentiation [ˌɛkspənɛnʃɪˈeɪʃ(ə)n] – зведення в ступінь

abstract algebra – абстрактна алгебра

abelian group – абелева група; комутативна група

cyclic group – циклічна група

commutative ring – комутативне кільце

equality of expressions – рівність виразів

integral domain – область цілісності

field of fractions – поле приватних, поле відносин

number field – числове поле; поле чисел

subring – підкільце (підмножина кільця)

absolute value – абсолютна величина, абсолютна величина, модуль (числа)

remainder – 1) залишок (від розподілу); 2) різницю; 3) залишковий член (ряду)

greatest common divisors – найбільший спільний дільник principal ideal domain – область головних ідеалів

fundamental theorem of arithmetic – основна теорема арифметики

totally ordered set – цілком упорядкована безліч

Exercise 2. Write the transcription and translate the following words. Make sentences with them.

decimal fraction

decimal point

power

denominator d

whole number

fractional part

to skip

calculations

to annex

repeating decimal

indefinitely

subtrahend

divisor

addend

multiplicand o

to separate

remainder

Exercise 3. Read and translate the text

A "construction" is drawing geometric figures with a high degree of accuracy. The construction performed constitutes both a proof of the existence of a geometric object and the solution of the problem.

The ancient Greeks were convinced that all plane figures can be constructed with a compass and a straightedge alone. Their methods of bisecting a line and an angle are ingenious and hard to improve on. They worked with all numbers geometrically. A length was chosen to represent the number 1, and all other numbers were expressed in terms of this length. They solved equations with unknowns by series of geometric constructions. The answers were line segments whose length were the unknown value sought.

The Greeks imposed the restrictions of straightedge and compass for the construction of the problems. It is supposed that this tradition was started by Plato Greece's greatest philosopher. He claimed that more complicated instruments called for manual skill unworthy of a thinker.

The Greeks failed to obtain the solution of the famous problems under the restrictions specified not due to the lack of ingenuity of the geometers. The Greeks' persistent efforts to find compass-and-straightedge ways of trisecting an angle, squaring the circle and duplicating the cube were not futile for almost 2000 years. The Greeks made great math discoveries on the way.

The desire to gain full understanding of the theoretical character of the problems inspired many great mathematicians- among them Descartes, Gauss, Poncelet, Lindemann – to mention but a few. The long years of labor on these "impractical", "worthless" problems indicate the care, patience, persistence and rigor of mathematicians in their attempts to perform the constructions and justify them theoretically. The problems did not exhaust themselves. Even nowadays some authors of the scientific papers issued "solutions" containing some fallacies. The search for the rigorous solution resulted in great discoveries and novel developments in mathematics.

Exercise 4. Which of these statements are true? Correct the false ones.

1. The ancient Egyptians were convinced that all plane figures can be constructed with a compass and a straightedge alone. 2. The Greeks succeeded to obtain the solution of the famous problems under the restrictions specified not due to the lack of ingenuity of the geometers. 3. The problems did not exhaust themselves. 4. The construction constitutes a proof of the existence of a geometric object. 5. The Greeks imposed the restrictions of straightedge and compass for the construction of the problems

Exercise 5. Translate the sentences into English.

1. Відрізки, що з'єднують вершину конуса з точками кола основи, що називаються утворюючими конуса.

2. Обсяг будь-якої трикутної піраміди дорівнює одній третині твору площі основи на висоту.

3. Оссю правильної піраміди називається пряма, що містить її висоту.

4. Ребра призми, що з'єднують вершини основ, називаються бічними ребрами.

5. Багатогранником називається тіло, обмежене кінцевим числом площин,

6. Бічна поверхня прямої призми дорівнює добутку периметра основи на висоту призми.

7. Пірамідою називається геометрична фігура з багатокутним основою та чотирма сторонами у вигляді трикутників, що сходяться у вершині піраміди.

8. Протилежні сторони паралелограма рівні та паралельні.

9. Конус – це тверде тіло з однією вершиною та основою у вигляді площини.

10. Правильною вважається піраміда з віссю, перпендикулярною до основи.

11. Конуси, призми та піраміди названі за типом їх основ.

12. Висотою конуса називається перпендикулярна відстань від його вершини до площини основи.

Exercise 6. Translate the following sentences into English.

1. У разі обидві теореми – як пряма, і зворотна – виявляються справедливими. 2. П'ять аксіом Евкліда – це пропозиції, що вводять відносини рівності чи нерівності величин. 3. Підручник Евкліда з геометрії «Початку» читали, читають і читатимуть багато людей. 4. Пропозиція, яка випливає безпосередньо з аксіоми, називається слідством. 5. Наступні дві теореми обернені один до одного. 6. Одна і та ж пропозиція може бути або не бути істинною щодо іншої множини припущень. 7. У будь-якій теоремі є дві

частини: гіпотеза та висновок. 8. Вас просять записати коротко припущення, що ви зробили. 9. Аксіома – це справжнє, вихідне положення теорії. 10. Постулат – це твердження, яке у будь-якій наукової теорії як істинне, хоч і доведене її засобами, і тому він грає у ній роль аксіоми.

Exercise 7. Match the terms from the left column and the definitions from the right column:

fraction	the number or quantity by which the dividend is divided to produce the quotient
expression	any quantity expressed in terms of a numerator and denominator
divisor	a showing by a symbol, sign, figures
dividend	the term above or to the left of the linee in a fraction
common factor	the term below or to the right of the line in a fraction
numerator	to change in denomination or form without changing in value
denominator	factor common to two or more numbers
to reduce	the number or quantity to be divided

Exercise 8. Match the terms from the left column and the definitions from the right column:

algebra	a number or quantity be subtracted from another one
to add	to take away or deduct (one number or quantity from another)
addition	the result obtained by adding numbers or quantities
addend	the amount by which one quantity differs from another

to subtract	to join or unite (to) so as to increase the quantity, number, size, etc. or change the total effect
subtraction	a number or quantity from which another is to be subtracted
subtrahend	equal in quantity value, force, meaning
minuend	an adding of two or moree numbers to get a number called the sum
equivalent	a mathematical system using symbols, esp. letters, to generalize certain arithmetical operations and relationships

Exercise 9. Form verbs from the following nouns and adjectives by adding prefix "en" – ("em"). Translate them into Ukrainian. Example: large – enlarge

1. rich 2. danger 3. chain 4. code 5. circle 6. courage 7. gender 8. body 9. force 10. frame 11. list 12. trap 13. trust 14. title 15. cipher 16. brace 17. act 18. place

Exercise 10. Form words with opposite meaning by adding prefix "dis" -. Translate them into Ukrainian.

Example: armament – disarmament

1. ability 2. agreement 3. proportion 4. accord 5. appear 6. advantage 7. belief 8. balance 9. credit 10. inherit 11. courage 12. illusion 13. join 14. honor 15. persuade 16. regard 17. remember 18. trust 19. qualify 20. similar 21. colour.

Exercise 11. Translate the following words and word-combinations into English:

1. довірити 2. кодувати 3. збагачувати 4. породжувати 5. втілювати 6. нездатність 7. недолік 8. розхолоджувати 9. порушення рівноваги 10. втілювати 11. забувати.

Exercise 12. Match these terms with their definitions.

1. geometric progression 2. product 3. integer 4. power 5. logarithm	a) any rational number that can be expressed as the sum or difference of a finite number of units b) (also called: exponent, index) a number or variable placed as a superscript to the right of another number or quantity indicating the number of times the number or quantity is to be multiplied by itself c) the result of the multiplication of two or more numbers, quantities etc. d) a sequence of numbers, each of which differs from the succeeding one by a constant ratio e) the exponent indicating the power to which a fixed number, the base, must be raised to obtain a given number or variable

Exercise 13. Read and translate the text

A decimal fraction is a special type of fraction written without a denominator (which is 10 or a power of 10) but in which the number of figures on the right-hand side of a dot, called the decimal point, indicates whether the denominator is 10 or a higher power of 10; e.g. 2/10 is written as a decimal in the form 0.2, 23/100 as 0.23, and 23/1000 as 0.023.

If any figure of the number is moved one place to the left, the value of the number is multiplied by 10. Suppose you have been given 587.9 where 9 has been separated from 587 by a point, but not a comma (a comma separates each group or period of numbers). The numeral 587 names a whole number. The sign (.) is called a decimal point. All digits to the left of the decimal point represent whole numbers. All digits to the right of the decimal point represent fractional parts of 1.

The digits to the right of the decimal point name the numerator of the fraction, and the number of such digits is the denominator. The place-value position at the right of the ones place is called tenth. You obtain a tenth by dividing 1 by 10. Such numerals like 687.9 are called decimals.

You read .2 as two tenth. To read .0054 you skip two zeroes and say fifty four ten thousands. Decimals like .777…, or 0.242424…, are called repeating decimals. In a repeating decimal the same numeral or the same set of numerals is repeated over and over again indefinitely. In our development of rational numbers we have named them by fractional numerals.

We know that rational numerals can just as well be named by decimal numerals. As you might expect, calculations with decimal numerals give the same results as calculations with the corresponding fractional numerals.

Decimal fractions are added in the same way that the whole numbers are added. Since only like decimals fractions can be added, that is hundredths to hundredths, the tenths to tenths, the addends are arranged in a vertical column with the decimal points directly below one another, all the way down to the answer. In subtracting decimal fractions we must write decimal fractions so that the decimal point of the minuend, subtrahend and remainder are below each other. Zeroes should be annexed so that both minuend and subtrahend are carried out to the same number of places.

Check the answer the same way you check the subtraction of whole numbers. In multiplying a decimal fraction or mixed decimals multiply as you do whole numbers. Then, starting at the right, mark off as many decimal places in the product as there are in the multiplier and multiplicand together. To divide a number by 10 or any power of ten, move the decimal point in the dividend as many places to the left as there are zeroes in the divisor. Add zeroes when needed

Exercise 14. Discuss the question. 1. Why do we need logarithms now that we have computers?

Exercise 15. Be ready to speak on: 1. The broad subdivision of mathematics. 2. The study of quantity. 3. The modern study of space. 4. Calculus as a powerful tool to investigate change. 5. The structural properties of mathematical objects. 6. Artificial intelligence

Exercise 16. Translate the following sentences into English: Основні підрозділи математики виникли як відповідь на нагальні потреби: вести підрахунки у торгівлі, вимірювати землю, передбачувати астрономічні події та розуміти зв'язок між числами. Оці чотири потреби розвинулись в математиці в окреме вивчення кількісних (арифметика), просторових (геометрія), змінних (алгебра) та структурних (аналіз) відношень. Окрім того, досліджуються стосунки математики з іншими дисциплінами: логікою, теорією множин та емпіричною математикою, застосовною до різних наук (прикладною математикою). Основні характеристики цілих чисел досліджуються у так званій теорії чисел. По мірі того, як у подальшому розвивалась система чисел, окрім цілих чисел увагу привернули так звані раціональні числа (дробі).

Exercise 17. Fill in the gaps using a modal + have + past participle.

a. Algebraic formulas for finding the volumes of cylinders and sphere ……………… (be used) in Ancient Egypt to compute the amount of grain contained in them. b. The discovery of the theorem of Pythagoras ……………… (hardly make) by Pythagoras himself, but it was certainly made in his school. c. Regardless of what mystical reasons ………………(motivate) the early Pythagorean investigators, they discovered many curious and fascinating number properties. d. Imaginary numbers …………… (be looking) like higher magic to many eighteenth century mathematicians. e. The symbol ……………… (be used) in the sixteenth century and it resembled a manuscript form of the small r (radix), the use of the symbol for square root had become quite standard. f. Descartes' geometric representation of negative numbers …………… (help) mathematicians to make negative numbers more acceptable.

Exercise 18. Rewrite these sentences following the model to make them more natural.

Example: To say no to people is hard → It's hard to say no to people.

a. To select two points on a line, labeling them and referring to the line in this way is more convenient.

b. To use colored chalk is more effective.

c. To memorize all of these relations is very difficult.

d. To distinguish the elements of a set from the "non elements" is very essential.

e. To point out that elements of a set need not be individual, but may themselves be sets is very important.

f. To determine the exact image in that case is impossible.

g. To have a more simplified system of notation is desirable.

h. To see that the meaning of an expression, depending on its context, is very clear.

i. To give your full name is compulsory.

j. To do the measuring as accurately as possible is very necessary.

k. To cut MN in two or three parts is permissible.

Exercise 19. Fill in the gaps using the correct form of the verb in brackets.

All calls ……………… (register) by the Help Desk staff. Each call …………… (evaluate) and then ………… (allocate) to the relevant support group. If a visit ………… (require), the user ……………… (contact) by telephone, and an appointment ……………(arrange). Most calls ………… (deal with) within one working day. In the event of a major problem requiring the removal of a user's PC, a replacement can usually …………. (supply).

Exercise 20. Make the sentences passive. Use "by …" only if it is necessary to say who does / did the action.

a. Charles Babbage designed a machine which became the basis for building today's computer in the early 1800s.

b. People submerged geometry in a sea of formulas and banished its spirit for more than 150 years.

c. People often appreciate analytical geometry as the logical basis for mechanics and physics.

d. Bill Gates founded Microsoft.

e. People call the part of the processor which controls data transfers between the various input and output devices the central processing unit (CPU).

f. You may use ten digits of the Hindu–Arabic system in various combinations. Thus we will use 1, 2 and 3 to write 123, 132, 213 and so on.

g. Mathematicians refer to a system with which one coordinates numbers and points as a coordinate system or frame of reference.

h. People similarly establish a correspondence between the algebraic and analytic properties of the equation f (x, y) = 0, and geometric properties of the associated curve.

i. In 1946 the University of Pensylvania built the first digital computer.

Exercise 21. Change the following passive sentences into active.

a. This frame of reference will be used to locate a point in space.

b. Although solid analytic geometry was mentioned by R. Descartes, it was not elaborated thoroughly and exhaustively by him.

c. Most uses of computers in language education can be described as CALL.

d. Since many students are considerably more able as algebraists than as geometers, analytic geometry can be described as the "royal road" in geometry that Euclid thought did not exist.

e. Now new technologies are being developed to make even better machines.

f. Logarithm tables, calculus, and the basis for the modern slide rule were not invented during the twentieth century.

g. After World War 2 ended, the transistor was developed by Bell Laboratories.

h. The whole subject matter of analytic geometry was well advanced, beyond its elementary stages, by L. Euler.

Exercise 22. Speak on the Topic "Decimal Numerals", give your own examples.

Exercise 23. Read and translate the text

Decimal fractions are the fractions in which the denominator (y in the image) must be 10 or a multiple of 10 like 100, 1000, 10000, and so on. The numerator can be any integer (between -infinity and +infinity). These decimal fractions are usually expressed in decimal numbers (numbers with a decimal point).

In algebra, a decimal fraction is a number having 10 or the powers of 10 like 10^1, 10^2, 10^3, and so on in the denominator.

Examples of Decimal Fractions

- 7/10000 is a decimal fraction written in the decimal form as 0.0007.
- 19/10 is a decimal fraction written in the decimal form as 1.9.
- 39/1000 is a decimal fraction written as 0.039.

Non Examples of Decimal Fractions

Other fractions with non-ten numbers in the denominator are not decimal fractions. They are:

- 37/8
- 2/1083
- 83/145

Reading Decimal Fractions

Let us consider a scenario where 1 is in the numerator. We will consider different denominators to understand how these terms are read with this numerator.

- 1/10 is read as one-tenth.
- 1/100 is read as one-hundredth.
- 1/1000 is read as one-thousandth.

When the value of the numerator is more than one, we add an ‘s’ to the name. So, for instance, 3/10 is read as three-tenths.

History of Decimal Fractions

The Chinese first developed and used decimal fractions at the end of the 4th century BC, which spread to the Middle East before reaching Europe.

Conversion to Decimal Fractions

1. Conversion from fractions to decimal fractions:

- Let us consider an example of a fraction, 3/2.
- The first step would be to consider the number that gives 10 or a multiple of 10 when multiplied by the denominator. In this case, 5 multiplied by 2 gives 10.
- Now multiply the numerator and denominator with the same number to get your decimal fraction. Here, 3 x 5/ 2 x 5 gives 15/10.
- Thus, the decimal fraction of 3/2 is 15/10.

2. Conversion from mixed numbers to decimal fractions:

- Convert the mixed fraction into a normal fraction.
- Follow the steps for converting fractions to decimal fractions.

3. Conversion from decimal numbers to decimal fractions:

- Write the original decimal number in the numerator and denominator form by placing 1 in the denominator: 4.3/1.
- For every space that you move the decimal point, add a zero next to the 1 in the denominator: 43/10 (As we can see one shift of decimal space, one 0 must be added to the denominator).

4.3/1

43.0/10

- Once the number in the numerator is non-decimal, you have got your decimal fraction: 4.3 = 43/10.

Real-Life Application of Decimal Fractions

Decimal fractions are used for understanding precise quantities instead of whole numbers. You will also use them for expressing percentages. For instance, 97% can be written as 97/100 for ease of calculation.

Here are some scenarios where you might encounter decimal fractions:

- Coins (They are a fraction of Rupees)
- Weighing products
- Measuring ingredients while cooking

Decimal fractions encourage students to learn about precise quantities. This will help them understand weights like 3.2 kg and distances like 7.85 km. The first step towards a better understanding of decimal numbers is practicing decimal fraction problems every day. The idea of taking a pen and paper to solve sums is dull and uninteresting for students. They need entertaining ways to entice them towards practicing the sums.

Exercise 24. Answer the following questions:

1.What is the decimal fraction? 2. How do we write decimal fractions? 3. How do you compare decimal fractions? 4. How do you change decimal fractions? 5. How are decimal fractions added? 6. How do we write decimal fractions when we want to subtract them? 7. How do we check the answer? 8. How do we multiply decimal fractions? 9. How do we divide decimal fractions? 10. How do we arrange numbers in adding decimal fractions?

Individual Work

Task 1. Over to you 1. Web research task. Find out as much as you can about Numerical analysis and its applications. Web search key words: P vs NP, unsolved problems, Clay Institute, RSA cryptosystem, binary notation.

Solving Decimal Word Problems

Problem 1: If 58 out of 100 students in a school are boys, then write a decimal for the part of the school that consists of boys.

Problem 2: Five swimmers are entered into a competition. Four of the swimmers have had their turns. Their scores are 9.8 s, 9.75 s, 9.79 s, and 9.81 s. What score must the last swimmer get in order to win the competition?

Problem 3: To make a miniature ice cream truck, you need tires with a diameter between 1.465 cm and 1.472 cm. Will a tire that is 1.4691 cm in diameter work? Explain why or why not.

Problem 4: Ellen wanted to buy the following items: A DVD player for $49.95, a DVD holder for $19.95 and a personal stereo for $21.95. Does Ellen have enough money to buy all three items if she has $90 with her?

Problem 5: Melissa purchased $39.46 in groceries at a store. The cashier gave her $1.46 in change from a $50 bill. Melissa gave the cashier an angry look. What did the cashier do wrong?

Task 2. Follow this link. **https://www.youtube.com/watch?v=XNnRKAwwKWc.** What is Decimal Number & Decimal Point? In this educational math video for kids, you'll learn the definition of a decimal and how it relates to numbers. Watch the video What is Decimal Number & Decimal Point? *Math for Kids* as many times as you need. Write out 10 topical theses. Prepare your own presentation on the thema What is the decimal number and decimal point?

Task 3. Follow this link

https://www.youtube.com/watch?v=9frz4ODJUc0. Watch the video about

Decimals as many times as you need. Write out 10 topical theses**.** Write down the main points and tell about it.

Task 4. Follow this link https://www.youtube.com/watch?v=kwh4SD1ToFc. Watch the video Math Antics - Decimal Arithmeticas many times as you need. Write out 10 topical theses. Prepare your own presentation on the thema Math Antics - Decimal Arithmetic.

Task 5. Follow this link https://ed.ted.com/lessons/a-brief-history-of-numerical-systems-alessandra-king. 1, 2, 3, 4, 5, 6, 7, 8, 9... and 0. With just these ten symbols, we can write any rational number imaginable. But why these particular symbols? Why ten of them? And why do we arrange them the way we do? Alessandra King gives a brief history of numerical systems. Watch the video A brief history of numerical systems as many times as you need. Make sure that you are throughall stages: ”Think”, “Dig Deeper” and “Discuss”. All necessary translations and thoughts you are welcome to take down into your copy-book.

Task 6. Follow this link **https://www.youtube.com/watch?v=xiMuFg9UqNY.**

A simple, clear, visual definition of "decimal". Watch the video What is a Decimal? as many times as you need. Write out 10 topical theses. Write down the main points and tell about it.

UNIT 6. GEOMETRY

Exercise 1. Read and memorise the following words and word combinations.

Infinitely large or infinitely small elements – нескінченно великі та нескінченно малі елементи (частини)

magnitude – величина; абсолютне значення; модуль

ordered group – упорядкована група

ordered field – впорядковане поле

local field – локальне поле

infinitesimal – нескінченно мала величина

linearly ordered group – лінійно впорядкована група

Archimedean group – архімедова група

p-adic numbers – p-адичні числа

absolute value – абсолютна величина, абсолютне значення

ultrametric property – ультраметрична властивість

axiomatic theory – аксіоматична теорія

least upper bound property – властивість точної верхньої межі

bounded above – обмежений зверху

proof by contradiction – доказ від протилежного

Exercise 2. Write the transcription and translate the following words. Make sentences with them.

point line angle point of intersection

angular point

straight

ray pencil of rays

curved line

right angle

reflex angle

acute angle

obtuse angle

corresponding angle

adjacent angle

supplementary angle

complementary angle

interior angle

exterior angle

plane triangle

equilateral triangle

isosceles triangle

acute-angled triangle

obtuse-angled triangle

right-angled triangle

quadrilateral square

rectangle

rhombus rhomboid trapezium

deltoid irregular quadrilaterals

polygon

circle

center

circumference (periphery)

diameter

semicircle

radius

tangent

point of contact

secant chord

segment

arc

sector

ring (annulus) concentric circles

axis of coordinates

axis of abscissae

axis of ordinate

values of abscissae and ordinates

conic section

parabola – branches of paraвol

vertex of paraвol

ellipse

foci of the ellipse

transverse axis (major axis)

hyperbola

asymptote

solids

cube

plane surface (plane)

edge

parallelepiped

triangular prism

cylinder circular plane

sphere cone

Exercise 3. Read the text and put the paragraphs into the correct order.

Engineers, architects and people of many other professions use lines and figures in their daily work. The study of lines and closed figures made by lines is called geometry. Geometry is the branch of mathematics which investigates the relations, properties and measurements of solids, surfaces, lines and angles.

Geometry is a very old subject. It probably began in Babylonia and Egypt. Men needed practical ways for measuring their land, for building pyramids, and for defining volumes. The Egyptians were mostly concerned with applying geometry to their everyday problems. Yet, as the knowledge of Egyptians spread to Greek found the ideas about geometry very intriguing and mysterious.

The Greek began to ask “Why? Why is that true?” In 300 B. C. all the known facts about Greek geometry were put into a logical sequence by Euclid. His book called Elements, is one of the most famous books of mathematics. In recent years men have improved on Euclid’s work. Today geometry includes not only the study of the shape and size of the earth and all things on it, but also the study of relations between geometric objects.

The most fundamental idea in the study of geometry is the idea of a point. We will not try to define what a point is, but instead discuss some of its properties. Think of a point as an exact location in space. You cannot see a point, feel a point, or move a point, because it has no dimensions.

There are points (locations) on the earth, in the earth, on the sun, and everywhere in space. When writing about points, you represent the points by dots. Remember the dot is only a picture of a point and not the point itself. Points are commonly referred to by using capital letters.

Exercise 4. Give the English equivalents of the following Ukrainian words and word combinations:

Вичитуване, величина, зменшуване, алгебраїчне додавання, еквівалентне вираження, віднімати, різницю, додавання, складати, доданок, сума, числове, числа зі знаками, відносні числа, поділ, множення, ділити, залишок, приватний, твір, вираз, зворотна операція, дільник, ділимо, множник, множимо, суми, знак множення, знак розподілу.

Exercise 5. Translate the text into English.

Додаванням у математиці називається дію (operation), що виконується над двома числами, іменованими (named) складовими, щоб одержати шуканого числа, суми. Дане дію можна визначити збільшення величини одного числа інше число.

При додаванні двох дробів необхідно привести обидва дроби до спільного знаменника, а потім скласти чисельники. Додавання чисел одночасно і комутативно, і асоціативно. При складанні комплексних чисел треба складати дійсну (real) і уявну (imaginary) частини окремо (separately).

Дія, зворотне (inverse to) додавання в математиці називається відніманням. Це процес, у якому дані два числа і потрібно знайти третє, шукане число. При цьому, при додаванні шуканого третього числа одного з даних, має вийти друге з даних чисел.

Exercise 6. Match these terms with their definitions.

1. prime number 2. composite number 3. magnitude 4. divisor 5. factorization 6. factor	a) a number or quantity to be divided into another number or quantity (the dividend) b) a number assigned to a quantity and used as a basis of comparison for the measurement of similar quantities c) an integer that cannot be factorized into other integers but is only divisible by itself or 1 d) a positive integer that can be factorized into two or more other positive integers e) one of two or more integers or polynomials whose product is a given integer or polynomial f) the decomposition of an object (for example, a number, a polynomial, or a matrix) into a product of other objects, or factors, which when multiplied together give the original

Exercise 7. Form abstract nouns from the following nouns and adjectives by changing final "t" ("te") into -"cy" -. Translate them into Ukrainian.

Example: urgent – urgency

1. excellent 2. private 3. accurate 4. intimate 5. obstinate 6. illiterate 7. delicate 8. autocrat 9. aristocrat 10. decent 11. secret 12. democrat 13. efficient 14. emergent 15. despondent 16. adequate 17. candidate

Exercise 8. Form abstract nouns from the following verbs by adding suffixes -"ance" or -"ence". Translate them into Ukrainian.

Example: attend – attendance

1. enter 2. admit 3. ignore 4. inherit 5. exist 6. disturb 7. accord 8. accept 9. perform 10. confer 11. diverge 12. excel.

Exercise 9. Match these terms with their definitions.

1. chord 2. arc 3. angle 4. trigonometry) 5. sine	a) a section of a curve, graph, or geometric figure b) a straight line connecting two points on a curve or curved surface c) the space between two straight lines that diverge from a common point or between two planes that extend from a common line d) the branch of mathematics concerned with the properties of trigonometric functions and their application to the determination of the angles and sides of triangles. Used in surveying and navigation e) a trigonometric function that in a right-angled triangle is the ratio of the

	length of the opposite side to that of the hypotenuse

Exercise 10. Match the terms from the left column and the definitions from the right column:

A

algebra	a number or quantity be subtracted from another one
to add	to take away or deduct (one number or quantity from another)
addition	the result obtained by adding numbers or quantities
addend	the amount by which one quantity differs from another
to subtract	to join or unite (to) so as to increase the quantity, number, size, etc. or change the total effect
subtraction	a number or quantity from which another is to be subtracted
subtrahend	equal in quantity value, force, meaning
minuend	an adding of two or moree numbers to get a number called the sum
equivalent	a mathematical system using symbols, esp. letters, to generalize certain arithmetical operations and relationships

B

to divide	to test, measure, verify or control by investigation, comparison or examination
division	the process of finding the number or quantity (product) obtained by repeated additions of a specified number or quantity
dividend	the number by which another number is multiplied
divisor	what is left undivided when one number is divided by another that is not one of its factors
to multiply	to separate into equal parts by a divisor
multiplication	the process of finding how many times a number is contained in another number
multiplicand	the number or quantity to be divided
multiplier	the quantity obtained by multiplying two or more quantities together
remainder	the number that is multiplied by another
product	the number or quantity by which the dividend is divided to produce the quotient
to check	to find the product by multiplication

Exercise 11. Read and translate the text

Geometry

Geometry (Greek; geo = earth, metria = measure) arose as the field of knowledge dealing with spatial relationships. For the ancient Greek mathematicians, geometry was the crown jewel of their sciences, reaching a completeness and perfection of methodology that no other branch of their knowledge had attained.

They expanded the range of geometry to many new kinds of figures, curves, surfaces, and solids; they changed its methodology from trial-and-error to logical deduction; they recognized that geometry studies “external forms”, or abstractions, of which physical objects are only approximations; and they developed the idea of an “axiomatic theory” which, for more than 2000 years, was regarded to be the ideal paradigm for all scientific theories.

The Muslim mathematicians made considerable contributions to geometry, trigonometry and mathematical astronomy and were responsible for the development of algebraic geometry.

The 17th century was marked by the creation of analytic geometry, or geometry with coordinates and equations, associated with the names of Rene Descartes and Pierre de Fermat.

In the 18th century, differential geometry appeared, which was linked with the names of L. Euler and G. Monge.

In the 19th century, Carl Frederich Gauss, Janos Bolyai and Nikolai Ivanovich Lobachevsky, each working alone, created non-Euclidean geometry. Euclid’s fifth postulate states that through a point outside a given line, it is possible to draw only one line parallel to that line, that is, one that will never meet the given line, no matter how far the lines are extended in either direction.

But Gauss, Bolyai and Lobachevsky demonstrated the possibility of constructing a system of geometry in which Euclid’s postulate of the unique parallel was replaced by a postulate stating that through any point not on a given straight line an infinite number of parallels to the given line could be drawn. Their works influenced later researchers, including Riemann and Einstein.

Exercise 12. Answer the following questions.

1. What is the origin of the term geometry? 2. What does geometry deal with? 3. What was the contribution of Greek mathematicians to the science of geometry? 4. Who contributed to the development of algebraic geometry? 5. Who was analytic geometry created by? 6. Whose names was differential geometry associated with? 7. Whose names was the creation on non-Euclidean geometry linked with? 8. Whose works were later influenced by non-Euclidean geometry?

Exercise 13. Discussion point What is the role of Ptolemy's theorem in trigonometry? Discuss this question in small groups of three or four. Choose a person from your group for a brief summary of your discussion

Exercise 14. The present simple or the past simple. Put the verbs in brackets in the correct forms.

The problem of constructing a regular polygon of nine sides which ………… (require) the trisection of a 600 angle ……… (be) the second source of the famous problem. The Greeks ……… (add) "the trisection problem" to their three famous unsolved problems. It ……… (be) customary to emphasize the futile search of the Greeks for the solution. The widespread availability of computers …………… (have) in all, probability changed the world for ever. The microchip technology which …………(make) the PC possible has put chips not only into computers, but also into washing machines and cars. Fermat almost certainly ………… (write) the marginal note around 1630, when he first ………… (study) Diophantus's Arithmetica. I ………… (protest) against the use of infinitive magnitude as something completed, which ……… (be) never permissible in maths, one ………… (have) in mind limits which certain ratio ……….. (approach) as closely as desirable while other ratios may increase indefinitely (Gauss). In 1676 Robert Hooke …………….(announce) his discovery concerning springs. He ……………..(discover) that when a spring is stretched by an increasing force, the stretch variesdirectly according to the force.

Exercise 15. Use the correct form of the degrees of comparison.

1) We all use this method of research because it is ………………. (interesting) the one we followed. 2) I could solve quicker than he because the equation given to me was……….(easy) the one he was given. 3) The remainder in this operation of division is …………….. (great) than 1. 4) The name of Leibnitz is …………….. (familiar) to us as that of Newton. 5) Laptops are ………… (powerful) microcomputers. We can choose either of them. 6) A mainframe is……… (large) and………….(expensive) a microcomputer. 7) One of the ……… (important) reasons why computers are used so widely today is that almost

every big problem can be solved by solving a number of little problems. 8) Even the …………(sophisticated) computer, no matter how good it is, must be told what to do.

Exercise 16. Rewrite the sentences putting the prepositions in front of or at the end of the relative clauses.

a. She works for a company. It has a very good reputation. The company……………………………………………………………...

b. After the great impetus given to the subject by R. Descarter and P. Fermat, we find analytical geometry in a form. With the form we are familiar today. After the great impetus given to the subject by these two men, ……………………………………………………………………………...

c. Most systems have a special area of the screen. On the screen icons appear. Most systems have a special area of the screen ……………………………………………………………………………. d. The salesperson was correct in saying that goods must be returned to the store. From there they were purchased. The salesperson was correct in saying that goods must be returned to the store …………………………………………………………….…………..

e. I deal with customers. Most of them are very pleasant. Most of the customers………………………………………………………

f. The simplest problem of tracing polar curves is the case. There is only one value of θ in this case. The simplest problem……………………………………………….………..

Exercise 17. Put the verbs in brackets in the correct form.

a. If we …………… (consider) the third example, we……………… (see) that the magnitude of the common ratio was less than 1. b. If we …………… (assume) the geometric mean of two numbers to be the square root of their product, what ……… the geometric mean between 2 and 8 ………… (be)? c. If the system ………… (crash), we…………… (lose) all our latest data. d. If paradoxes ……… (be not) so subtle and colourful by Zeno's, mathematicians ………………… (not pay attention) to them. e. If you ……….. (use) a monitor with interlaced video for word processing, you …………… (not use) a standard. f. If Godel's incompleteness theorem …… (be not proved), rigorous and consistent philosophy of maths ……. (be created) in the 20th century.

Exercise 18. Make sentences from the following notes.

a. experiment / would / have / give / more / reliable / results – it / have / be prepared / with / greater care. ……………………………………………………………………………….

b. we / be able / start / this project / two / month – board / think / it / be / good idea. ……………………………………………………………………………….

c. he / would / have / read / his paper – he / have / be given / time. ……………………………………………………………………………….

d. I / will / send / you / fax - you / get / all information / you / need / today. ……………………………………………………………………………….

e. whole thing / would not / have happened – they / have / be more careful. ……………………………………………………………………………….

f. one / can / readily / find / length / third / side – one / know / length / two / side / triangle / and / measure / angle / between / they. ………………………………………………………………………………..

Exercise 19. Put in the correct form of the verbs.

a. Emma went into the sitting room. It was empty, but the television was still on. Some one ………………… (watch) it. b. I ……………… (play) tennis, so I had a shower. I was annoyed because I …………… (not win) a single game. c. The walkers finally arrived at their destination. They …………… (walk) all day, and they certainly needed a rest. They ……….. (walk) thirty miles. d. I found the calculator. I ……………… (look for) it for ages. e. I finally bought a new calculator. I …………… (look) everywhere for the old one.

Exercise 20. Speaking. Answer the following questions:

1. Is geometry an old subject? 2. What is geometry? 3. Did geometry begin in England? 4. Were Egyptians mostly concerned with the practical use of geometry?

5. Did the knowledge of Egyptians spread to Greece? 6. Is Euclid's book called Elements famous? 7. Does geometry include only the study of the shape and size objects? 8. Is the idea of a point fundamental in geometry? 9. Can one feel, see, move or hold a point? 10. Has a point any dimensions? 11. How do we represent a point in geometry? 12. Are points represented by dots? 13. How many lines can be drawn through one point? 14. What is a segment? 15. Does a line segment include its endpoints? 16. Can you draw a straight line by using a ruler? 17. How many lines can be drawn between two points? 18. What kind of lines do you know?

Exercise 21. Read and translate the text

Geometry, the branch of mathematics concerned with the shape of individual objects, spatial relationships among various objects, and the properties of surrounding space. It is one of the oldest branches of mathematics, having arisen in response to such practical problems as those found in surveying, and its name is derived from Greek words meaning "Earth measurement." Eventually it was realized that geometry need not be limited to the study of flat surfaces (plane geometry) and rigid three-dimensional objects (solid geometry) but that even the most abstract thoughts and images might be represented and developed in geometric terms.

This article begins with a brief guidepost to the major branches of geometry and then proceeds to an extensive historical treatment. For information on specific branches of geometry, *see* Euclidean geometry, analytic geometry, projective geometry, differential geometry, non-Euclidean geometries, and topology.

Major branches of geometry

Euclidean geometry

In several ancient cultures there developed a form of geometry suited to the relationships between lengths, areas, and volumes of physical objects. This geometry was codified in Euclid's *Elements* about 300 BCE on the basis of 10 axioms, or postulates, from which several hundred theorems were proved by deductive logic. The *Elements* epitomized the axiomatic-deductive method for many centuries.

Analytic geometry

Analytic geometry was initiated by the French mathematician René Descartes (1596–1650), who introduced rectangular coordinates to locate points and to enable lines and curves to be represented with algebraic equations. Algebraic geometry is a modern extension of the subject to multidimensional and non-Euclidean spaces.

Projective geometry

Projective geometry originated with the French mathematician Girard Desargues (1591–1661) to deal with those properties of geometric figures that are not altered by projecting their image, or "shadow," onto another surface.

Differential geometry

The German mathematician Carl Friedrich Gauss (1777–1855), in connection with practical problems of surveying and geodesy, initiated the field of differential geometry. Using differential calculus, he characterized the intrinsic properties of curves and surfaces. For instance, he showed that the intrinsic curvature of a cylinder is the same as that of a plane, as can be seen by cutting a cylinder along its axis and flattening, but not the same as that of a sphere, which cannot be flattened without distortion.

Non-Euclidean geometries

Beginning in the 19th century, various mathematicians substituted alternatives to Euclid's parallel postulate, which, in its modern form, reads, "given a line and a point not on the line, it is possible to draw exactly one line through the given point parallel to the line." They hoped to show that the alternatives were logically impossible. Instead, they discovered that consistent non-Euclidean geometries exist.

Topology

Topology, the youngest and most sophisticated branch of geometry, focuses on the properties of geometric objects that remain unchanged upon continuous deformation—shrinking, stretching, and folding, but not tearing. The continuous development of topology dates from 1911, when the Dutch mathematician L.E.J. Brouwer (1881–1966) introduced methods generally applicable to the topic.

History of geometry

The earliest known unambiguous examples of written records—dating from Egypt and Mesopotamia about 3100 BCE—demonstrate that ancient peoples had already begun to devise mathematical rules and techniques useful for surveying land areas, constructing buildings, and measuring storage containers. Beginning about the 6th century BCE, the Greeks gathered and extended this practical knowledge and from it generalized the abstract subject now known as geometry, from the combination of the Greek words *geo* ("Earth") and *metron* ("measure") for the measurement of the Earth.

In addition to describing some of the achievements of the ancient Greeks, notably Euclid's logical development of geometry in the *Elements*, this article examines some applications of geometry to astronomy, cartography, and painting from classical Greece through medieval Islam and Renaissance Europe. It concludes with a brief discussion of extensions to non-Euclidean and multidimensional geometries in the modern age.

Ancient geometry: practical and empirical

The origin of geometry lies in the concerns of everyday life. The traditional account, preserved in Herodotus's *History* (5th century BCE), credits the Egyptians with inventing surveying in order to reestablish property values after the annual flood of the Nile. Similarly, eagerness to know the volumes of solid figures derived from the need to evaluate tribute, store oil and grain, and build dams and pyramids. Even the three abstruse geometrical problems of ancient times—to double a cube, trisect an angle, and square a circle, all of which will be discussed later—probably arose from practical matters, from religious ritual, timekeeping, and construction, respectively, in pre-Greek societies of the Mediterranean. And the main subject of later Greek geometry, the theory of conic sections, owed its general importance, and perhaps also its origin, to its application to optics and astronomy.

While many ancient individuals, known and unknown, contributed to the subject, none equaled the impact of Euclid and his *Elements* of geometry, a book now 2,300 years old and the object of as much painful and painstaking study as the Bible. Much less is known about Euclid, however, than about Moses. In fact, the only thing known with a fair degree of confidence is that Euclid taught at the Library of Alexandria during the reign of Ptolemy I (323–285/283 BCE). Euclid wrote not only on geometry but also on astronomy and optics and perhaps also on mechanics and music. Only the *Elements*, which was extensively copied and translated, has survived intact.

Euclid's *Elements* was so complete and clearly written that it literally obliterated the work of his predecessors. What is known about Greek geometry before him comes primarily from bits quoted by Plato and Aristotle and by later mathematicians and commentators. Among other precious items they preserved are some results and the general approach of Pythagoras (*c*. 580–*c*. 500 BCE) and his followers. The Pythagoreans convinced themselves that all things are, or owe their relationships to, numbers. The doctrine gave mathematics supreme importance in the investigation and understanding of the world. Plato developed a similar view, and philosophers influenced by Pythagoras or Plato often wrote ecstatically about geometry as the key to the interpretation of the universe. Thus ancient geometry gained an association with the sublime to complement its earthy origins and its reputation as the exemplar of precise reasoning.

Exercise 22. What does the quotation mean? What is your interpretation or opinion of it? "A man is like a fraction whose numerator is what he is and whose denominator is what he thinks of himself. The larger the denominator, the smaller the fraction." ~ Laura Surtees

Individual Work

Task 1. Over to you. Prepare the project. Math Matters in Everyday Life.

Task 2. Project. Visit the website http://discovermagazine.com/tags/math, make notes about one of the articles, which you enjoyed reading the most and bring to the next lesson to share with your groupmates.

Task 3. Follow this link https://ed.ted.com/lessons/how-many-ways-are-there-to-prove-the-pythagorean-theorem-betty-fei. What do Euclid, 12-year-old Einstein, and American President James Garfield have in common? They all came up with elegant proofs for the famous Pythagorean theorem, one of the most fundamental rules of geometry and the basis for practical applications like constructing stable buildings and triangulating GPS coordinates. Betty Fei details these three famous proofs. Watch the video as many times as you need. Make sure that you are throughall stages: ”Think”, “Dig Deeper” and “Discuss”. All necessary translations and thoughts you are welcome to take down into your copy-book.

Task 4. Follow this link https://ed.ted.com/best_of_web/FoTYJ95H. The team from Numberphile printed one million decimal places of Pi onto a piece of paper which stretched for over a mile. They rolled it out on a runway usually used for testing planes and cars. Write down the main points and tell about it.

Task 5. Follow this link https://ed.ted.com/lessons/euclid-s-puzzling-parallel-postulate-jeff-dekofsky. Euclid, known as the "Father of Geometry," developed several of modern geometry's most enduring theorems--but what can we make of his mysterious fifth postulate, the parallel postulate? Jeff Dekofsky shows us how mathematical minds have put the postulate to the test and led to larger questions of how we understand mathematical principles. Watch the video as many times as you need. Make sure that you are throughall stages: ”Think”, “Dig Deeper” and “Discuss”. All necessary translations and thoughts you are welcome to take down into your copy-book.

UNIT 7. FOURIER'S METHOD

Exercise 1. Read and memorise the following words and word combinations

Series – ряд; прогресія, послідовність

term – член, елемент

terms of a sequence – члени (елементи) послідовності

infinite sequence – нескінченна послідовність

summation – підсумовування

Zeno's dichotomy – дихтомія Зенона

formula (pl. formulae, formulas) – формула (мн. формули)

algorithm – алгоритм

formal sum – формальна сума

converge – 1) сходитися; прагнути до (загальної) межі; 2) зводити (в одну точку)

diverge – 1) розходитися; 2) відхилятися (від лінії, напряму)

index set – індексна безліч

recurring decimal – періодичний десятковий дріб

completeness property – властивість повноти

geometric series – геометричний ряд, нескінченна геометрична прогресія

constant – константа, постійна (величина)

Exercise 2. Write the transcription and translate the following words. Make sentences with them.

circle

transcendental number

rectangle

ratio

circumference

indivisible unit

odd number

irrational number

even number

pentagon

polygon

Exercise 3. Read and translate the text

In its most general form Fourier's method represents a signal, determined by a function f, as a combination of waves of all possible frequencies. This is called the Fourier transform of the wave. It replaces the original signal by its spectrum: a list of amplitudes and frequencies for the component sines and cosines, encoding the same information in a different way – engineers talk of transforming from the time domain to the frequency domain.

When data are represented in different ways, operations that are difficult or impossible in one representation may become easy in the other. For example, you can start with a telephone conversation, form its Fourier transform, and strip out all parts of the signal whose Fourier components have frequencies too high or too low for the human ear to hear. This makes it possible to send more conversations over the same communication channels, and it's one reason why today's phone bills are, relatively speaking, so small. You can't play this game on the original, untransformed signal, because that doesn't have 'frequency' as an obvious characteristic. You don't know what to strip out.

One application of this technique is to design buildings that will survive earthquakes. The Fourier transform of the vibrations produced by a typical earthquake reveals, among other things, the frequencies at which the energy imparted by the shaking ground is greatest. A building has its own natural modes of vibration, where it will resonate with the earthquake, that is, respond unusually strongly.

So the first sensible step towards earthquake-proofing a building is to make sure that the building's preferred frequencies are different from the earthquake's. The earthquake's frequencies can be obtained from observations; those of the building can be calculated using a computer model. The Fourier transform has become a routine tool in science and engineering; its applications include removing noise from old sound recordings, such as clicks caused by scratches on vinyl records, finding the structure of large biochemical molecules such as DNA using X-ray diffraction, improving radio reception, tidying up photographs taken from the air, sonar systems such as those used by submarines, and preventing unwanted vibrations in cars at the design stage.

Fourier analysis has become a reflex among engineers and scientists, but for some purposes the technique has one major fault: sines and cosines go on forever. Fourier's method runs into problems when it tries to represent a compact signal. It takes huge numbers of sines and cosines to mimic a localised blip.

The problem is not getting the basic shape of the blip right, but making everything outside the blip equal to zero. You have to kill off the infinitely long rippling tails of all those sines and cosines, which you do by adding on even more high-frequency sines and cosines in a desperate effort to cancel out the unwanted junk. So the Fourier transform is hopeless for blip-like signals: the transformed version is more complicated, and needs more data to describe it, than the original.

(an extract from the book The story of mathematics by Ian Stewart)

Exercise 4. Give the English equivalents of the following Ukrainian words and word combinations:

Тотожність, перестановка, корінь, рішення, невідома величина, основа, умовне рівняння, ступінь, показник ступеня, висловлювання (формулювання), еквівалентна операція, тотожне рівняння, рівняння з одним невідомим, рівняння першого ступеня, підстановка, підкорене вираз, лінійне рівняння.

Exercise 5. Translate the following sentences into English.

1. Математика як наука складається з таких областей як арифметика, алгебра, геометрія, математичний аналіз і т.д.

2. Математичний вираз x + 3 = 8 - це рівняння, що показує що x + 3 та 8 рівні. Таким чином, вважається, що рівняння – це символічне висловлювання, що показує рівність двох або більше математичних виразів.

3. Рівняння типу x + 3 = 8 містить одне невідоме.

4. Щоб вирішити рівняння; необхідно виконати певні математичні операції, такі як додавання та віднімання, множення та розподіл.

5. Розв'язати рівняння означає знайти значення невідомих, які задовольняють рівняння.

6. Рівняння – це вираз рівності між двома величинами.

7. Всі рівняння 2-го, 3-го та 4-го ступеня вирішуються в радикалах.

8. Лінійне рівняння може бути записане у формі 3x+2=12.

Exercise 6. Translate the text into English.

Трикутники.

Випуклий трикутник називається правильним, якщо всі його сторони рівні та рівні всі його кути.

Багатокутник називається вписаним у коло, якщо всі його вершини лежать на деякому колі. Багатокутник називається описаним біля кола, якщо всі його сторони стосуються цього кола.

Правильний опуклий багатокутник є одночасно вписаним у коло та описаним біля неї.

Кутом опуклого багатокутника при певній вершині називається кут, утворений його сторонами, які сходяться на цій вершині. Зовнішнім кутом опуклого багатокутника при цій вершині називається кут, суміжний із внутрішнім кутом багатокутника при цій вершині.

Exercise 7. Match these terms with their definitions.

1. permutation 2. group 3. symmetry 4. a quadratic	a) an exact correspondence in position or form about a given point, line, or plane b) an ordered arrangement of the numbers, terms etc., of a set into specified groups

5. resolvent 6. to reduce 7. a quintic 8. a radical 9. translation 10. reflection 11. commutative law	c) an equation containing one or more terms in which the variable is raised to the power of two, but no terms in which it is raised to a higher power d) a set that has an associated operation that combines any two members of the set to give another member and that also contains an identity element and an inverse for each element e) to modify or simplify the form of (an expression equation), esp. by substitution of one term by another f) something that resolves; solvent g) of, relating to, or containing roots of numbers or quantities h) of or relating to the fifth degree j) a transformation in which the direction of one axis is reversed or which changes the sign of one of the variables k) a transformation in which the origin of a coordinate system is moved to another position so that each axis retains the same direction or, equivalently, a figure or curve is moved so that it retains the same orientation to the axes l) (of an operator) giving the same result irrespective of the order

Exercise 8. Form verbs from the following adjectives and nouns by adding suffix - "en". Example: threat - threaten

1. short 2. wide 3. strength 4. length 5. deep 6. soft 7. weak 8. hard 9. white 10. like 11. broad.

Exercise 9. Form nouns from the following verbs by adding suffixes "tion" ("sion"), "ence" ("ance"), "ment".

1. depend 2. agree 3. intensify 4. constitute 5. explain 6. achieve 7. estrange 8. express 9. admit 10. converge 11. diverge 12. expand 13. pollute 14. create 15. perform 16. multiply 17. exist 18. expect 19. assume 20. deduce 21. attach 22. amend 23. improve 24. ignore.

Exercise 10. Match the terms from the left column and the definitions from the right column:

abscissa	the vertical Cartesian coordinate on a plane, measured from the ***x***-axis along a line parallel with the ***y***-axis to point P
axis (axes)	perpendicular, or at a right angle, to the plane of the horizon; upright, straight up or down, etc.
Cartesian coordinates	a) of or pertaining to a point on a surface at which the curvature is zero, b) of or lying in the plane
horizontal	any of the four parts formed by rectangular coordinate axes on a plane surface
oblique	the horizontal Cartesian coordinate on a plane, measured from the ***y***-axis to point P
ordinate	a) a straight line through the center of a plane figure or a solid, esp. one around which the parts are symmetrically arranged, b) a straight line for measurement or reference, as in a graph
origin	with its axes not perpendicular to its base
planar	parallel to the plane of the horizon, not vertical
quadrant	in a system of Cartesian coordinates, the point at which the axes intersect; base point where the abscissa and the ordinate equal zero
vertical	a pair of numbers that locate a point by its distances from two fixed, intersecting, usually perpendicular lines in the same plane

Exercise 11. Read and translate the text

The Development of Mathematics in the 17th Century The scientific revolution of the 17th century spurred advances in mathematics as well. The founders of modern science – Nicolaus Copernicus, Johannes Kepler, Galileo, and Isaac Newton – studied the natural world as mathematicians, and they looked for its mathematical laws. Over time, mathematics grew more and more abstract as mathematicians sought to establish the foundations of their fields in logic.

The 17th century opened with the discovery of logarithms by the Scottish mathematician John Napier and the Swiss mathematician Justus Byrgius. Logarithms enabled mathematicians to extract the roots of numbers and simplified many calculations by basing them on addition and subtraction rather than on multiplication and division.

Napier, who was interested in simplification, studied the systems of the Indian and Islamic worlds and spent years producing the tables of logarithms that he published in 1614. Kepler's enthusiasm for the tables ensured their rapid spread. The 17th century saw the greatest advances in mathematics since the time of ancient Greece. The major invention of the century was calculus.

Although two great thinkers - Sir Isaac Newton of England and Gottfried Wilhelm Leibniz of Germany – have received credit for the invention, they built on the work of others.

As Newton noted, "If I have seen further, it is by standing on the shoulders of giants." Major advances were also made in numerical calculation and geometry. Gottfried Leibniz was born (1st July, 1646) and lived most of his life in Germany. His greatest achievement was the invention of integral and differential calculus, the system of notation which is still in use today. In England, Isaac Newton claimed the distinction and accused Leibniz of plagiarism, that is stealing somebody else's ideas but stating that they are original.

Modern-day historians, however, regard Leibniz as having arrived at his conclusions independently of Newton. They point out that there are important differences in the writings of both men. Differential calculus came out of problems of finding tangents to curves, and an account of the method is published in Isaac Barrow's "Lectiones geometricae" (1670). Newton had discovered the method (1665-66) and suggested that Barrow include it in his book.

Leibniz had also discovered the method by 1676, publishing it in 1684. Newton did not publish his results until 1687. A bitter dispute arose over the priority for the discovery. In fact, it is now known that the two made their discoveries independently and that Newton had made it ten years before Leibniz, although Leibniz published first. The modern notation of dy/dx and the elongated s for integration are due to Leibniz.

The most important development in geometry during the 17th century was the discovery of analytic geometry by Rene Descartes and Pierre de Fermat, working in- dependently in France. Analytic geometry makes it possible to study geometric figures using algebraic equations. By using algebra, Descartes managed to overcome the limitations of Euclidean geometry. That resulted in the reversal of the historical roles of geometry and algebra.

The French mathematician Joseph Louis Lagrange observed in the 18th century, "As long as algebra and geometry proceeded along separate paths, their advance was slow and their applications limited. But when these sciences joined company, they drew from each other fresh vitality and thenceforward marched on at a rapid pace toward perfection." Descartes' publications provided the basis for Newton's mathematical work later in the century.

Pierre de Fermat, however, regarded his own work on what became known as analytic geometry as a reformulation of Appollonius's treatise on conic sections. That treatise had provided the basic work on the geometry of curves from ancient times until Descartes.

Exercise 12. Answer the following questions.

1. What scholars are considered to be the founders of modern science? 2. Why did mathematics grow more and more abstract? 3. Who were logarithms discovered by? 4. What did logarithms enable mathematicians to do? 5. What was the major invention of the 17th century? 6. What is the essence of analytic

geometry? 7. Why did a dispute arise between Leibniz and Newton? 8. What enabled Descartes to overcome the limitations of Euclidean geometry? 9. Whose publications provided the basis for Newton's mathematical work later in the century?

Exercise 13. What does the quotation mean? What is your interpretation or opinion of it? "Mathematics, in one view, is the science of infinity." ~P. Davis and R. Hersh

Exercise 14. Turn from Active into Passive.

Model: 1. Scientists introduce new concepts by rigorous definitions. New concepts are introduced by rigorous definitions. 2. Mathematicians cannot define some notions in a precise and explicit way. Some notions cannot be defined in a precise and explicit way.

1. People often use this common phrase in such cases. 2. Even laymen must know the foundations, the scope and the role of mathematics. 3. In each country, people translate mathematical symbols into peculiar spoken words. 4. All specialists apply basic symbols of mathematics. 5. You can easily verify the solution of this equation. 6. Mathematicians apply abstract laws to study the external world of reality. 7. A mathematical formula can represent interconnections and interrelations of physical objects. 8. Scientists can avoid ambiguity by means of symbolism and mathematical definitions. 9. Mathematics offers an abundance of unsolved problems. 10.Proving theorems and solving problems form a very important part of studying mathematics. 11.At the seminar, they discussed the recently published article. 12.They used a mechanical calculator in their work. 13. One can easily see the difference between these machines. 14.They are checking the information. 15.The researchers have applied new methods of research. 16.They will have carried out the experiment by the end of the week.

Exercise 15. Put the adjective or adverb in brackets in the necessary degree of comparison.

1. The scholar's (significant) contribution to mathematics was his discovery of analytic geometry. 2. Diophantus' book was on (high) level than the works of

Egyptian and Babylonian mathematics. 3. (early) records of organized mathematics date back to ancient times. 4. (simple) types of calculators could give results in addition and subtraction only. 5. (often used) numbers were two and three. 6. For numbers (large) than two and three, different word-combinations were used. 7. Even (primitive) people were forced to count and measure. 8. In the 19th century, mathematics was regarded (much) as the science of relations. 9. Mathematics is said to be (close) to art than to science. 10. Mathematics becomes the science of relations and structure in (broad) sense.

Exercise 16. Give English equivalents to the following words and wordcombinations:

1. Натуральне число 2. множина 3. натуральний ряд 4. антиномія 5. аксіоматизація 6. фінітна математика 7. арифметична ієрархія 8. обмеження розміру 9. ряд кардинальних чисел 10. зведення елімінації 11. доказово-теоретичний ряд 12. рекурсивна теорія 13. доказ логічності (послідовності) 14. аксіома заміни 15. аксіома вибору 16. теоретико-множинні уявлення 17. теоретико множинні антиномії 18. теоретико-множинна мова.

Exercise 17. Turn direct speech into indirect (reported) speech.

a. Plato advised, “The principal men of our state must go and learn arithmetic, not as amateurs, but they must carry on the study until they see the nature of numbers with the mind only.” b. Descartes, father of modernism, said, “All nature is a vast geometrical system. Thus all the phenomena of nature are explained and some demonstration of them can be given.” c. In Descartes’ words, “You give me extension and motion then I’ll construct the universe.” d. The often repeated motto on the entrance to Plato’s Academy said, “None ignorant of geometry enter here.” e. J. Kepler affirmed: “The reality of the world consists of its maths relations. Maths laws are true cause of phenomena. ” f. I. Newton said, “ I don’t know what I may appear to the world; but to myself I seem to have been only like a boy playing on the seashore, and diverting myself now and then by finding a smoother pebble or a prettier shell than usual; whist the great ocean of truth lay all undiscovered before me. If I saw a little farther than others, it is because I stood on the shoulders of giants

Exercise 18. Complete the sentences using a gerund as an attribute. 1. I didn't very much like the idea of … . 2. What is the purpose of … ? 3. She had no difficulty (in) … . 4. You have made great progress in … . 5. He was late, and he was afraid of … . 6. Can you imagine the pleasure of … . 7. He always produces the impression of … . 8. I am afraid you do not realize the importance of … .

Exercise 19. Ask the special questions. 1. Some properties are established by way of reasoning (how). 2. Geometry is concerned with the properties and relationships of figures in space (what ... with). 3. Some figures such as cubes and spheres have three dimensions (how many). 4. Many discoveries were made in the nineteenth century (when). 5. The truth of nonmathematical propositions in real life is much less certain (where). 6. The given proposition and its converse can be stated as follows (in what way). 7. Pure mathematics deals with the development of knowledge for its own purpose and need (what ... with). 8. Carl Gauss proved that every algebraic equation had at least one 21 root (who). 9. There are three words having the same meaning (how many). 10. The given definition corresponds to the idea of uniqueness (what).

Exercise 20. Discussion point What if you were to give a short presentation on Euclid's most important contribution to mathematics? What would you say?

Exercise 21. Read and translate the text

You don't have to be around signal processing very long before you realize that Fourier methods play a huge role in the field. Newcomers often wonder why they are so important. There are several very good reasons for the prominence of Fourier methods in signal processing. They offer substantial intuition, naturally follow from the way the physical world interacts with signals, and are amazingly useful for computation.

Fourier Methods in Signal Processing

There are multiple Fourier methods that are used in signal processing. The most common are the Fourier transform, the discrete-time Fourier transform, the discrete Fourier transform, and the short-time Fourier transform. Fourier methods are used for two primary purposes: mathematical analysis of problems and numerical analysis of data. The Fourier transform and discrete-time Fourier transform are mathematical analysis tools and cannot be evaluated exactly in a computer. The Fourier transform is used to analyze problems involving continuous-time signals or mixtures of continuous- and discrete-time signals. The discrete-time Fourier transform is used to analyze problems involving discrete-time signals or systems. In contrast, the discrete Fourier transform is the computational workhorse of signal processing. It is used solely for numerical analysis of data. Lastly, the short-time Fourier transform is a variation of the discrete Fourier transform that is used for numerical analysis of data whose frequency content changes with time.

You may be wondering why I haven't mentioned the fast Fourier transform algorithm or FFT. The FFT is not a distinct Fourier method, but is an efficient computational technique for evaluating the discrete Fourier transform.

Fourier methods are named after Joseph Fourier, a French mathematician and physicist that lived from 1768 to 1830. He pioneered the use of sinusoids for representing arbitrary functions. An important problem of his day was understanding heat flow. Heat flow is governed by a partial differential equation called the heat equation. Fourier developed a technique for solving partial differential equations that assumed the solution was given by a weighted sum of harmonically related sinusoids. This work was controversial at the time and initially rejected by Lagrange, Laplace, and Legendre. Fourier persisted with his approach and his methods are now widely used in mathematics, science, and, of course, signal processing.

Frequency Intuition

All Fourier methods use sinusoids of different frequencies as building blocks to represent arbitrary signals. The notion of frequency is widely understood. We often talk about high frequency and low frequency sounds. We are comfortable with the idea of different musical instruments having different frequency ranges, and with a complex musical piece being obtained by combining instruments. A musical score is very much like a Fourier decomposition in that it describes the music in terms of individual notes with different frequencies. These general concepts make it relatively easy to transition to the idea of representing an arbitrary signal as a sum of sinusoids of different frequencies.

Spectral analysis is an important aspect of signal processing. It is analogous to producing a score from a piece of music. The goal is to start with a signal and identify the strength of the sinusoidal components that make up the signal. The strength or amplitude of the sinusoids are displayed as a function of frequency. The importance of spectral analysis lies in the intuitive understanding it provides of the signal. For example, the figure at left depicts the amplitude of sinusoids as a function of frequency for measured sunspot activity from 1700 to 1987. We see a dominant component with frequency slightly less than 0.1 cycles per year, which suggests a periodic pattern of about 11 years in the activity.

Exercise 22. What does the quotation mean? What is your interpretation or opinion of it? “But think of Adam and Eve like an imaginary number, like the square root of minus one: you can never see any concrete proof that it exists, but if you include it in your equations, you can calculate all manner of things that couldn't be imagined without it.” ~ Philip Pullman

Individual Work

Task 1. Over to you. Visit the website https://theconversation.com/us/topics/mathematics98, make notes about one of the articles, which you enjoyed reading the most and bring to the next lesson to share with your groupmates.

Task 2. Project ideas:

- all about circles
- pyramids
- Pythagoras
- parabolas
- surveying activities in the field
- the Earth from a mathematical viewpoint
- the surface area of the school building
- outstanding mathematicians
- mathematics and the arts
- fractals: beauty and chaos in mathematics

Task 3. Follow this link. https://www.ted.com/talks/arthur_benjamin_the_magic_of_fibonacci_numbers. Math is logical, functional and just ... awesome. Mathemagician Arthur Benjamin explores hidden properties of that weird and wonderful set of numbers, the Fibonacci series. (And reminds you that mathematics can be inspiring, too!). Watch the video about The magic of Fibonacci numbers as many times as you need. Write down the main points and tell about it.

Task 4. Follow this link https://ed.ted.com/lessons/why-can-t-you-divide-by-zero.

In the world of math, many strange results are possible when we change the rules. But there's one rule that most of us have been warned not to break: don't divide by zero. How can the simple combination of an everyday number and a basic operation cause such problems? Watch the video Why can't you divide by zero? as many times as you need. Make sure that you are throughall stages: ”Think”, “Dig Deeper” and “Discuss”. All necessary translations and thoughts you are welcome to take down into your copy-book.

Task 5. Follow this link https://ed.ted.com/best_of_web/VvOg8aiS.

Rubik's Cube pro Collin Burns tells us how he solved the cube in 5.25 seconds. Watch the video How a 15-year-old solved a Rubik's Cube in 5.25 seconds as many times as you need. Make sure that you are throughall stages: ”Think”, “Dig Deeper” and “Discuss”. All necessary translations and thoughts you are welcome to take down into your copy-book.

UNIT 8. THE BASIC OPERATIONS OF ARITHMETIC

Exercise 1. Read and memorise the following words and word combinations.

Function – функція

relation – відношення

set – множина

inputs – вхідні (вступні) дані

permissible – допустимий

outputs – вихідні дані

input variable – вхідна величина, вхідна змінна

argument of the function – аргумент функції

objects of investigation – об'єкт дослідження

graph of the function – графік функції

inverse – зворотна величина; зворотний, протилежний

solution – рішення

differential equation – диференціальне рівняння

ordered pair – впорядкована пара

domain – область визначення

codomain – область значень (функції), кообласть

range – слово може позначати і область значень та вихідні дані

unambiguous word – однозначне слово

avoid ambiguity – уникати неоднозначності (неясності, двозначності)

image of the function, image domain – область відображення

function space – функціональний простір

real analysis – аналіз дійсних чисел

complex analysis – комплексний аналіз

functional analysis – функціональний анаиз triangle – трикутник

rectangle – прямокутник

hexagon – шестикутник; шестигранник

square – квадрат

linked – пов'язаний

mapped – відображений; відображається

value – значення, величина

integer – ціле число

polygon – багатокутник; багатогранник; полігон

vertice – вершина

four shapes times five colors – 4 фігури помножені на 5 кольорів

notation – позначення; форма запису

signum function - знакова функція

Exercise 2. Write the transcription and translate the following words. Make sentences with them.

of two dimensions (the plane)

spherical geometry

an equilateral triangle

climax

spatial

tetrahedron

octahedron

dodecahedron

icosahedron

an obtuse angle

an isosceles triangle

postulates

common notions

faces

Exercise 3. Read and translate the text

We cannot live a day without numerals. Numbers and numerals are everywhere. On this page you will see number names and numerals. The number names are: zero, one, two, three, four and so on. And here are the corresponding numerals: 0, 1, 2, 3, 4, and so on. In a numeration system numerals are used to represent numbers, and the numerals are grouped in a special way. The numbers used in our numeration system are called digits. In our Hindu-Arabic system we use only ten digits: 0, 1, 2, 3, 4. 5, 6, 7, 8, 9 to represent any number. We use the same ten digits over and over again in a place-value system whose base is ten. These digits may be used in various combinations. Thus, for example, 1, 2, and 3 are used to write 123, 213, 132 and so on.

One and the same number could be represented in various ways. For example, take 3. It can be represented as the sum of the numbers 2 and 1 or the difference between the numbers 8 and 5 and so on.

A very simple way to say that each of the numerals names the same number is to write an equation — a mathematical sentence that has an equal sign (=) between these numerals. For example, the sum of the numbers 3 and 4 equals the sum of the numbers 5 and 2. In this case we say: three plus four (3+4) is equal to five plus two (5+2). One more example of an equation is as follows: the difference between numbers 3 and 1 equals the difference between numbers 6 and 4. That is three minus one (3-1) equals six minus four (6-4). Another example of an equation is 3+5 = 8. In this case you have three numbers. Here you add 3 and 5 and get 8 as a result. 3 and 5 are addends (or summands) and 8 is the sum. There is also a plus (+) sign and a sign of equality (=). They are mathematical symbols.

Now let us turn to the basic operations of arithmetic. There are four basic operations that you all know of. They are addition, subtraction, multiplication and division. In arithmetic an operation is a way of thinking of two numbers and getting one number. We were just considering an operation of addition. An equation like 7-2 = 5 represents an operation of subtraction. Here seven is the minuend and two is the subtrahend. As a result of the operation you get five. It is the difference, as you remember from the above. We may say that subtraction is the inverse operation of addition since 5 + 2 = 7 and 7 - 2 = 5.

The same might be said about division and multiplication, which are also inverse operations.

In multiplication there is a number that must be multiplied. It is the multiplicand. There is also a multiplier. It is the number by which we multiply. When we are multiplying the multiplicand by the multiplier we get the product as a result. When two or more numbers are multiplied, each of them is called a factor. In the expression five multiplied by two (5×2), the 5 and the 2 will be factors. The multiplicand and the multiplier are names for factors.

In the operation of division there is a number that is divided and it is called the dividend; the number by which we divide is called the divisor. When we are dividing the dividend by the divisor we get the quotient. But suppose you are dividing 10 by 3. In this case the divisor will not be contained a whole number of times in the dividend. You will get a part of the dividend left over. This part is called the remainder. In our case the remainder will be 1. Since multiplication and division are inverse operations you may check division by using multiplication.

Exercise 4. Translate the following sentences into English.

1. У цій системі використовуються позитивні та негативні числа.

2. Позитивні та негативні числа представлені (to represent) відносинами цілих позитивних чисел.

3. Раціональні (rational) числа, своєю чергою, використовуються до створення ірраціональних (irrational) чисел.

4. У сукупності раціональні та ірраціональні числа становлять систему дійсних чисел.

5. Математичний аналіз — це розділ математики, що вивчає функції та межі.

6. Безліч Х є підмножиною іншої множини У тому випадку, якщо всі елементи множини Х одночасно є елементами множини У.

7. Аксіоми, що задовольняють безлічі дійсних чисел, можна умовно поділити на три категорії

Exercise 5. Translate the text into English.

Натуральні логарифми

Число е має дуже важливе значення (to be of great importance) у вищій математиці, його можна порівняти зі значенням Р геометрії. Число е застосовується як основа натуральних, або неперових лога-рифмів, що мають широке застосування (application) у математичному аналізі. Так, з їх допомогою багато формул можуть бути представлені в більш простому вигляді, ніж при користуванні десятковими логарифмами. Натуральний логарифм має символ.

Exercise 6. Match the terms from the left column and the definitions from the right column:

logarithm	to put (facts, statistics; etc.) in a table of columns
base	the decimal part of a logarithm to the base 10 as distinguished from the integral part called *the charac-teristic*
antilogarithm	a logarithm to the base *e*
characteristic	any number raised to a power by an exponent
mantissa	the exponent expressing the power to which a fixed number *(the base)* must be raised in order to produce a given number *(an antilogarithm)*
natural logarithm	the resulting number when a base is raised to power by a logarithm
to tabulate	the act of computing, calculation, b) a method of computing.

computation	the whole number, or integral part, of a logarithm as distinguished from the *mantissa*

Exercise 7. Read and write the numbers and symbols in full according to the way they are pronounced:

425 - 25 = 400

730 - 15=715

222 - 22 = 200

1617 + 17 = 1634

1215 + 60 = 1275

512 ÷ 8 = 64

1624 ÷ 4 = 406

456 ÷ 2 = 228

135 × 4 = 540

450 × 3 = 1350

34582 + 25814 = 60396

768903 - 420765 = 348138

1634986 - 1359251 = 275735

1000 ÷ 100 = 10

810 ÷ 5 = 162

5748 + 6238=11986

100 × 2 = 200

107 × 5 = 535

613 × 13 = 7969

1511 + 30 = 1541

755 × 4 ÷2 = 1510

123 ÷ 3 = 41

Exercise 8. Match the English words and word combinations with their Ukrainian equivalents.

1. the undefined term	a) вершина кута
2. to extend indefinitely	b) відмінні риси
3. the vertex of the angle	c) якщо не вказано інше
4. the interior of the angle	d) невизначений термін
5. distinguishing features	e) внутрішня частина кута
6. the exterior part	f) продовжуватися нескінченно
7. unless stated otherwise	g) зовнішня частина
8. reflex angle	h) суміжні кути
9. perpendicular bisector	i) кут між 180 та 360
10. adjacent angles	j) перпендикулярна бісектриса
11. intersecting lines	k) перпендикулярні прямі
12. parallel lines	l) прямі, що перетинаються
13. perpendicular lines	m) прямі прямі
14. acute angle	n) тупий кут
15. right angle	o) гострий кут
16. obtuse angle	p) прямий кут
17. straight angle	q) розгорнутий кут

Exercise 9. Form nouns from the following adjectives by adding suffix - "th" ("t"). Translate them into Ukrainian.

Example: wide – width

1. long 2. broad 3. strong 4. warm 5. deep 6. high

Exercise 10. Add to the following words (nouns, adjectives, adverbs, prepositions) suffix - "ward" to form adjectives or adverbs referring to direction or position. Translate them into Ukrainian.

Example: back – backward

1. lee 2. wind 3. sea 4. sun 5. star 6. down 7. up 8. to 9. in 10. out 11. for 12. on 13. home 14. way 15. west 16. south 17. east 18. north.

Exercise 11. Read and translate the text

During the 18th century, calculus became the cornerstone of mathematical analysis on the European continent. Mathematicians applied the discovery to a variety of problems in physics, astronomy, and engineering. In the course of doing so, they also created new areas of mathematics.

In France, Joseph Louis Lagrange made substantial contributions in all fields of pure mathematics, including differential equations, the calculus of variations, probability theory, and the theory of equations. In addition, Lagrange put his mathematical skills to work in the solution of practical problems in mechanics and astronomy.

The greatest mathematician of the 18th century, Leonard Euler of Switzerland, wrote works that covered the entire fields of pure and applied mathematics. He wrote major works on mechanics that preceded Lagrange's work. He won a number of prizes for his work on the orbits of comets and planets, the field known as celestial mechanics. But Euler is best known for his works in pure mathematics.

In one of his works, Introduction to the Analysis of Infinites, published in 1748, he approached calculus in terms of functions rather than the geometry of curves. Other works by Euler contributed to number theory and differential geometry (the application of differential calculus to the study of the properties of curves and curved spaces). Mathematicians succeeded in firming the foundations of analysis and discovered the existence of additional geometries and algebras and more than one kind of infinity.

The 19th century began with the German mathematician Carl Frederich Gauss. He ranks as one of the greatest mathematicians of the world. His book Inquiries into Arithmetic published in 1801 marks the beginning of modern era in number theory. Gauss called mathematics the queen of sciences and number theory the queen of mathematics.

Almost from the introduction of calculus, efforts had been made to supply a rigorous foundation for it. Every mathematician made some effort to produce a logical justification for calculus and failed. Although calculus clearly worked in solving problems, mathematicians lacked rigorous proof that explained why it worked.

Finally, in 1821, the French mathematician Augustin Louis Cauchy established a rigorous foundation for calculus with his theory of limits, a purely arithmetic theory. Later, mathematicians found Cauchy's formulation still too vague because it did not provide a logical definition of real number. The necessary precision for calculus and mathematical analysis was attained in the 1850s by the German mathematician Karl T. W. Weierstrass and his followers.

Another important advance in analysis came from the French mathematician Jean Baptiste Fourier, who studied infinite series in which the terms are trigonometric functions. Known today as Fourier series, they are still powerful tools in pure and applied mathematics.

The investigation of Fourier series led another German mathematician, Georg Cantor, to the study of infinite sets and to the arithmetic of infinite numbers. Georg Cantor began his mathematical investigations in number theory and went on to create set theory. In the course of his early studies of Fourier series, he developed a theory of irrational numbers.

Cantor and another German mathematician, Julius W. R. Dedekind, defined the irrational numbers and established their properties. These explanations hastened the abandonment of many 19th century mathematical principles. When Cantor introduced his theory of sets, it was attacked as a disease from which mathematics would soon recover. However, it now forms part of the foundations of mathematics. The application of set theory greatly advanced mathematics in the 20th century.

Exercise 12. Answer the following questions.

1. What did the discovery of calculus lead to? 2. What was Lagrange's contribution to pure and applied mathematics? 3. What did Euler's works contribute to? 4. What is the essence of differential geometry? 5. What event marked the beginning of modern era in number theory? 6. When was a rigorous foundation for calculus finally supplied? 7. What is the theoretical and practical

value of Fourier series? 8. What was Georg Cantor's contribution to mathematical studies? 9. Who were irrational numbers investigated and defined by? 10. What was the first reaction to Cantor's set theory? 11. Was the attitude to the discovery later changed?

Exercise 13. Discussion point Your Mathematics and Mechanics Faculty is trying to choose a new course for the coming year. The dean's office narrowed the list of suggestions down to two possibilities. You are part of the student committee that has been asked to recommend one of the courses. Course 1. The Lornz attractor. Course 2. Cellular automaton. Discuss these courses in small groups of three or four. Choose a person from your group for a brief summary of your discussion.

Exercise 14. Give English equivalents to the following words:

1. захищений (від вітру); 2. по вітру; 3. вперед; 4. всередину; 5. назовні; 6. поступовий (рух); 7. непокірний, загубивший шлях; 8. спрямований до зірок; 9. висота; 10. сила; 11. тепло; 12. глибина

Exercise 15. Grammar revision; the Continuous or Perfect Continuous Tenses.

1. I (to look for) a photographs my brother sent to me. 2. They (to have) a meeting now. 3. The phone always (to ring) when I (to have) a bath. 4. Friends always (to talk) to me when I (to try) to concentrate. 5. He (to watch) television when the door bell (to ring). 6. He (to build up) his business all his life. 7. They (to stay) with us for a couple of weeks. 8. By 1992 he (to live) there for ten years. 9. The video industry (to develop) rapidly. 10. He (to work) nights next week. 11. She (to spend) this summer in Europe. 12. Why are you so late? I (to wait) you for hours. 13. The boys must be tired. They (to play) football in the garden all afternoon. 14. The old town theatre is currently (to rebuild). 15. I usually (to go) to work by car, but I (to go) on the bus this week while my car (to repair).

Exercise 16. Complete these sentences by putting the verb in brackets into the Present Simple or the Present Continuous.

To solve the problem of gravitation, scientists …………… (consider) time-space geometry in a new way nowadays. Quantum rules …………… (obey) in any system. We …………… (use) Active Server for this project because it ……… (be) Webbased. Scientists …………… (trace and locate) the subtle penetration of quantum effects into a completely classical domain. Commonly we ………… (use) C + + and JavaScript. At the moment we …………… (develop) a Web–based project. Its domain ………… (begin) in the nucleus and ………… (extend) to the solar system. Right now I ………….. (try) to learn how to use Active Server properly

Exercise 17. Turn direct speech into reported speech.

1. Plato advised, "The principal men of our state must go and learn arithmetic, not as amateurs, but they must carry on the study until they see the nature of numbers with the mind only." 2. Descartes, father of modernism, said, "All nature is a vast geometrical system. Thus all the phenomena of nature are explained and some demonstration of them can be given." 3. In Descartes's words, "You give me extension and motion then I'll construct the universe." 4. The often repeated motto on the entrance to Plato's Academy said, "None ignorant of geometry enter here." 5. J. Kepler affirmed: "The reality of the world consists of its maths relations. Maths laws are true cause of phenomena. " 6. I. Newton said, "I don't know what I may appear to the world; but to myself I seem to have been only like a boy playing on the seashore, and diverting myself now and then by finding a smoother pebble or a prettier shell than usual; whist the great ocean of truth lay all undiscovered before me. If I saw a little farther than others, it is because I stood on the shoulders of giants".

Exercise 18. The present simple or the past simple. Put the verbs in brackets in the correct forms.

The problem of constructing a regular polygon of nine sides which …………. (require) the trisection of a 600 angle ……… (be) the second source of the famous problem. The Greeks ……… (add) “the trisection problem” to their three famous unsolved problems. It ……… (be) customary to emphasize the futile search of the Greeks for the solution. The widespread availability of computers …………… (have) in all, probability changed the world for ever. The microchip technology which …………(make) the PC possible has put chips not only into computers, but also into washing machines and cars. Fermat almost certainly ………… (write) the marginal note around 1630, when he first ………… (study) Diophantus's Arithmetica. I ………… (protest) against the use of infinitive magnitude as something completed, which ……… (be) never permissible in maths, one ………… (have) in mind limits which certain ratio ……….. (approach) as closely as desirable while other ratios may increase indefinitely (Gauss). In 1676 Robert Hooke.……………(announce) his discovery concerning springs. He …………….. (discover) that when a spring is stretched by an increasing force, the stretch variesdirectly according to the force.

Exercise 19. Change the following sentences using infinitive to express the purpose.

Model: We have to subtract this number from the sum obtained because we want to check the result of addition. – To check the result of addition, we have to subtract this number from the sum obtained.

1) We must know the details because we want to understand the situation. – 2) You must do the following because you want to operate this machine. – 3) He put the figures in a table because he wants to look at the data. – 4) He included the empty set at the beginning because he wants to have a complete table. 5) We made a conjecture and then proved this because we want to have the correct procedure.

Exercise 20. Discuss the following topics.

1. Aristotle's contribution to mathematical logic. 2. Development of mathematical logic in the 19th century. 3. Subfields of mathematical logic. 4. Mathematical logic and its importance for contemporary science.

Exercise 21. Read and translate the text

Basic Arithmetic Operations: The Four Fundamental Operators

Just like Julie Andrews told us in *The Sound of Music*, we should "start at the very beginning" because it's "a very good place to start." In math, we don't start with Do-Re-Mi, but we do build on the fundamentals known as arithmetic operations!

Mastering arithmetic operations means setting a strong foundation for a lifetime of successful math learning, so we strongly encourage taking the time to really commit to these skills!

If you're looking for more of a broad overview of the arithmetic branch of mathematics, let's take a step back so we can go through arithmetic as a whole first.

Ready to get started with arithmetic operations?

First of all, what is "arithmetic operations"?

Arithmetic operations are the building blocks for all mathematical processes and methods. (Yeah, they're kind of a big deal!) These types of operations are part of the "arithmetic" branch of math.

Arithmetic operations strip math down to the basics that we use every day, whether we realize it or not. Those basics are addition, subtraction, multiplication, and division.

Not so scary, right?

Basic arithmetic

You'll sometimes hear arithmetic operations referred to as "basic arithmetic," meaning the most fundamental mathematical operations.

Fundamental arithmetic operations

The fundamental arithmetic operations are typically thought to be addition, subtraction, multiplication, and division.

We'll dive into each more thoroughly in a moment!

Some schools will also include comparing numbers and evaluating powers (or exponents) as part of arithmetic operations. If you're not there yet, don't worry! Everyone moves at their own pace — and we can always help you when you do get there.

The four basic operations of math

Whether you're balancing your checkbook or ordering pizza for a party, chances are you're using some of the four basic arithmetic operations daily.

But sometimes, when something is so second-nature, it can be hard to explain it well. Here's a table of terms and examples that you can use when describing the four basic operations:

Operation	Verb	Example	Result vocabulary
Addition	Add	$1+1=2$	The result of addition is the "sum"
Subtraction	Subtract	$3-2=1$	The result of subtraction is the "difference"
Multiplication	Multiply	$4\times 2=8$ $2*3=6$ $5\cdot 2=10$	The result of multiplication is the "product"
Division	Divide	$12\div 3=4$ $10/2=5$	The result of division is the "quotient"

Now that we know more about each operation, we can drill down even further and take a look at each operation's operator.

What are arithmetic operators?

Arithmetic operators are the symbols we see in math problems that represent an action we should take. They're like little math GPS instructions, telling us what needs to happen in order for us to reach our final destination.

In other words, the operator tells us which operation to perform! For example, the operator tells us that we should subtract.

Let's look at each operator a little more closely:

Arithmetic operators: a guide

Learning arithmetic operators (and their related operations) is like learning to drive a car — you need know which pedal does what before you can hit the gas and start steering.

Here's a handy chart explaining what each operator means and how it might look on the page:

Operator	Operation
$+$	Addition
$-$	Subtraction
$\times, *, \cdot$	Multiplication
$\div, /$	Division

Exercise 22. What does the quotation mean? What is your interpretation or opinion of it? "Mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers, and we have reason to believe that it is a mystery into which the human mind will never penetrate." ~ Leonhard Euler

Individual Work

Task 1. Over to you. Visit the website https://www.nytimes.com/topic/subject/mathematics, make notes about one of the articles, which you enjoyed reading the most and bring to the next lesson to share with your groupmates.

Task 2. Project. Prepare a presentation on one of the topics:

• The Monty Hall Problem

• 0.999… = 1

• There are as many even numbers as natural numbers

• Benford’s Law

• The Birthday Paradox

• The Brocken Water Heater Problem

Task 3. Follow this link https://ed.ted.com/best_of_web/H8AVpKzt. Learn what an elliptical pool table can teach us about mathematics. Watch the video The Elliptical Pool Table: A Mathematician's Dream as many times as you need. Make sure that you are throughall stages: ”Think”, “Dig Deeper” and “Discuss”. All necessary translations and thoughts you are welcome to take down into your copy-book.

Task 4. Follow this link https://www.youtube.com/watch?v=U3tlaE8uY50. Every fraction can be written in its simplest form by dividing the numerator and denominator by their greatest common divisor. A rational number when expressed in terms of a simplest fraction with a positive denominator is said to be in standard form. Watch the video Standard Form | How to Determine Which Fraction is Greater as many times as you need. Write down the main points and tell about it.

Task 5. Follow this link https://ed.ted.com/lessons/can-you-find-the-next-number-in-this-sequence-alex-gendler. 1, 11, 21, 1211, 111221. These are the first five elements of a number sequence. Can you figure out what comes next? Alex

Gendler reveals the answer and explains how beyond just being a neat puzzle, this type of sequence has practical applications as well. Watch the video Can you find the next number in this sequence? as many times as you need. Make sure that you are throughall stages: ”Think”, “Dig Deeper” and “Discuss”. All necessary translations and thoughts you are welcome to take down into your copy-book.

UNIT 9. PERFECT NUMBERS

Exercise 1. Read and memorise the following words and word combinations.

Continuous function – безперервна функція

discontinuous function – дискретна, перервна функція

continuous inverse function – безперервна зворотна функція

homeomorphism – гомеоморфізм, топологічне відображення

height – висота, вершина, верх

open interval – відкритий інтервал

closed interval – замкнутий інтервал l

imit point – гранична точка

vacuously true – беззмістовно істинний

vacuous set – порожня множина

neighborhood – околиця (крапки)

metric topology – метрична топологія

infinitesimal – нескінченно мала (величина)

Non-standard analysis – нестандартний аналіз

Exercise 2. Write the transcription and translate the following words. Make sentences with them.

fractional component

set of integers

natural numbers

additive inverse

boldface

blackboard bold

countably infinite

algebraic number theory

algebraic integers

number line

unital ring

universal property

initial object

exponentiation

abstract algebra

abelian group

cyclic group

isomorphic

commutative monoid

integral domain

field of fractions

number field

чисел subring

absolute value

remainder

principal ideal domain

fundamental theorem of arithmetic

totally ordered set

ordered ring

discrete valuation ring

ordered pair of natural numbers

primitive data type

cardinality

natural numbers

Exercise 3. Read and translate the text

In mathematics the pursuit of perfection has led its aspirants to different places. There are perfect squares, but here the term is not used in an aesthetic sense. It's more to warn you that there are imperfect squares in existence. In another direction, some numbers have few divisors and some have many. But, like the story of the three bears, some numbers are 'just right'.

When the addition of the divisors of a number equals the number itself it is said to be perfect. The Greek philosopher Speusippus, who took over the running of the Academy from his uncle Plato, declared that the Pythagoreans believed that 10 had the right credentials for perfection.

Why? Because the number of prime numbers between 1 and 10 (namely 2, 3, 5, 7) equalled the non-primes (4, 6, 8, 9) and this was the smallest number with this property. Some people have a strange idea of perfection. It seems the Pythagoreans actually had a richer concept of a perfect number.

The mathematical properties of perfect numbers were delineated by Euclid in the Elements and studied in depth by Nicomachus 400 years later, leading to amicable numbers and even sociable numbers. These categories were defined in terms of the relationships between them and their divisors. At some point they came up with the theory of superabundant and deficient numbers and this led them to their concept of perfection.

Whether a number is superabundant is determined by its divisors and makes a play on the connection between multiplication and addition. Take the number 30 and consider its divisors, that is all the numbers which divide into it exactly and which are less than 30. For such a small number as 30 we can see the divisors are 1, 2, 3, 5, 6, 10 and 15. Totalling up these divisors we get 42. The number 30 is superabundant because the addition of its divisors (42) is bigger than the number 30 itself.

A number is deficient if the opposite is true – if the sum of its divisors is less than itself. So the number 26 is deficient because its divisors 1, 2 and 13 add up to only 16, which is less than 26. Prime numbers are very deficient because the sum of their divisors is always just 1. A number that is neither superabundant nor deficient is perfect. The addition of the divisors of a perfect number equal the number itself. The first perfect number is 6. Its divisors are 1, 2, 3 and when we add them up, we get 6. The Pythagoreans were so enchanted with the number 6 and the way its parts fitted together that they called it ‘marriage, health and beauty’. There is another story connected with 6 told by St Augustine (354–430). He believed that the perfection of 6 existed before the world came into existence and that the world was created in 6 days because the number was perfect

Exercise 4. Give the English equivalents of the following words and word combinations:

Тупий кут, розгорнутий кут, нульовий кут, кут піднесення, кут зниження, прямий кут, повний кут, сторона, напрям обертання, вершина, кут в межах від 180 ° 360, обертання (поворот), гострий кут, за годинниковою стрілкою, проти годинникової стрілки, кут обертання, що спрямовує кут.

Exercise 5. Translate the sentences into English.

1. Якщо дві сторони та кут між ними одного трикутника рівні відповідно двом сторонам та куту між ними іншого трикутника, то такі трикутники рівні.

2. Дві прямі називаються перпендикулярними, якщо вони перетинаються під прямим кутом.

3. Який кут називається прилеглим?

4. Доведіть, що вертикальні кути дорівнюють.

5. Сума цих трьох кутів дорівнює 270°.

Exercise 6. Match the terms from the left column and definitions from the right column:

an angle	formed by, or with reference to, a straight line or plane perpendicular to a base
null	of less than 90 degrees
right	designating an angle greater than a straight angle (180 degrees)
obtuse	height above a surface, as of the earth
flat	the shape made by two straight lines meeting at a

	common point, the vertex, or by two planes, meeting along an edge
acute	a decrease in force, activity, amount, etc. - a decrease in force, activity, amount, etc.
reflex	greater than 90 degrees and less than 180 degrees greater than 90 degrees and less than 180 degrees
elevation	designating of, or being zero, as: a) having all zero elements *(null* matrix), b) having a limit of zero *(null* sequence), c) having no members whatsoever *(null* set)
depression	absolute, positive

Exercise 7. Form new verbs by adding prefixes "over" – and "under"-. Example: to work – to overwork, to underwork.

1. to load 2. to act 3. to bid 4. to charge 5. to run 6. to praise 7. to do 8. to shoot 9. to value 10. to estimate 11. to feed.

Exercise 8. Translate into English.

1. Перша лінія, з якою ми знайомимося, вивчаючи математику, це пряма лінія. 2. Дати суворе визначення цього поняття дуже непросто. 3. У працях Евкліда (Euclid) лінія визначалася як довжина без товщини. 4. Кут – найпростіша геометрична фігура після точки, прямої, променя та відрізка. 5. Якщо в площині з точки провести два різні промені ОА та ОВ, то вони розділять площину на дві частини, кожна з яких називається кутом з вершиною О та сторонами ОА та ОВ. 6. Промінь, що ділить кут навпіл і бере початок у вершині кута, називається його бісектрисою. 7. Бісектриса розгорнутого кута ділить його на два суміжні кути, які називаються прямими кутами. 8. Велике значення для теорії та практики має визначення величини чи міри кута. 9. Основна властивість міри кута має полягати в тому, щоб рівні кути мали однакову міру. 10. Градусна міра використовується в елементарній геометрії для вимірювання кутів. 11. Кожен, напевно, знайомий

із транспортиром – вимірником кутів на кресленнях. 12. Кути менше прямого називаються гострими, а кути більше прямого, але менше розгорнутого, називаються тупими. 13. Перша книга Евкліда починається з 23 «визначень», у тому числі такі: точка є те, що немає частин; лінія є довжиною без ширини; лінія обмежена крапками; пряма лінія, однаково розташована щодо своїх точок; нарешті, дві прямі, що лежать в одній площині, називаються паралельними, якщо вони, як завгодно продовжені, не зустрічаються. 14. Виклад геометрії в «Початках» Евкліда вважалося зразком, якого прагнули слідувати вчені та за межами математики. 15. Цю теорему використовують, щоб показати, що експонентні функції, логарифмічні, тригонометричні функції безперервні. 16. Як основа логарифму зазвичай використовують число 10 або число е; відповідно говорять про десятковий (decimal logarithm) або про натуральний логарифм (natural logarithm).

Exercise 9. Form nouns from the following verbs by adding - "y".

Example: discover – discovery

1. recover 2. assemble 3. treasure 4. deliver 5. inquire 6. entreat 7. enter.

Exercise 10. Translate the following words into English

1. відкриття 2. переоцінити 3. недооцінити 4. переливатися через вінця 5. одужання 6. зібрання 7. розслідування 8. вхід 9. перевантажувати 10. постачання 11. скарбниця 12. прохання.

Exercise 11. Read and translate the text

Archimedes was the greatest mathematician, physicist and engineer of antiquity. He was born in the Greek city of Syracuse on the island of Sicily about 287 B.C. and died in 212 B. C. Roman historians have related many stories about Archimedes.

There is a story which says that once when Archimedes was taking a bath, he discovered a phenomenon which later became known in the theory of hydrostatics as Archimedes' principle. He was asked to determine the composition of the golden crown of the King of Syracuse, who thought that the goldsmith had mixed base metal with the gold.

The story goes that when the idea how to solve this problem came to his mind, he became so excited that he ran along the streets naked shouting “Eureka, eureka!” (“I have found it!”).

Comparing the weight of pure gold with that of the crown when it was immersed in water and when not immersed, he solved the problem. Archimedes was obsessed with mathematics, forgetting about food and the bare necessities of life. His ideas were 2000 years ahead of his time. It was only in the 17th century that his works were developed by scientists. There are several versions of the scientist’s death.

One of them runs as follows. When Syracuse was taken by the Romans, a soldier ordered Archimedes to go to the Roman general, who admired his genius. At that moment, Archimedes was absorbed in the solution of a problem. He refused to fulfill the order and was killed by the soldier.

Archimedes laid the foundations of mechanics and hydrostatics and made a lot of discoveries. He added new theorems to the geometry of the sphere and the cylinder and stated the principle of the lever. He also discovered the law of buoyancy.

Exercise 12. Answer the questions

1. When and where was Archimedes born? 2. How did he discover the famous principle known under his name in the theory of hydrostatics? 3. What was his emotional reaction to the solution of the problem? 4. What was Archimedes ordered to do when Syracuse was taken by the Romans? 5. Why did he refuse to fulfill the order? 6. What happened to him upon the refusal? 7. What were his contributions to science?

Exercise 13. What does the quotation mean? What is your interpretation or opinion of it? “You always admire what you really don't understand.” ~ Blaise Pascal

Exercise 14. Join the two sentences to make one sentence, beginning with a gerund.

Model: She's a teacher. It's hard work. Being a teacher is hard work / Teaching is hard work.

1. Capital letters are used to name geometrical objects. It is very convenient. 2. You are to classify these quadrilaterals. It requires the knowledge of some properties. 3. We are going to locate this point on the y axis. It will give us the first point on the line. 4. The student intends to divide a circle into a certain number of congruent parts. It will help him to obtain a regular polygon. 5. The base and the altitude of a rectangle are to be multiplied. It will give the product of its dimensions or the area of the rectangle. 6. Don't argue! It's no use. In a crossed quadrilateral, the interior angles on either side of the crossing add up to 720°. 7. Don't deny this fact! It is useless. A square is a quadrilateral, a parallelogram, a rectangle and a rhombus. 8. You are going to divide a heptagon (a 7-sided polygon) into five triangles. Is it any good?

Exercise 15. Complete each of the sentences below by choosing one of the pronouns in brackets.

1. ... arrived in good time and the meeting started promptly at 3.30 (anybody/ nobody/ everybody) 2. ... in the village went to the party but ... enjoyed it very much. (everybody/ no one/ some one), (anybody/ somebody/ nobody) 3. ... heard anything. (everyone/ nobody/ somebody) 4. "Who shall I give this one to? – You can give it to ... . It doesn't matter." (everyone/ nobody/ anybody) 5. That's a very easy job. ... can do it. (everyone/ nobody/ somebody). 6. Would you like ... to drink? (anything/ something/ nothing) 7. I thought I'd seen you ... . (anywhere/ somewhere/ nowhere) 8. There was ... to hide. (anywhere/ somewhere/ nowhere) 9. You still haven't told me ... . (anything/ something/ nothing) 10. Does ... agree with me? (anybody/ somebody/ nobody) 11. I want to introduce you to ... (no one/ someone/ any one) 12. The box was completely empty. There was ... in it. (nothing/ anything) 13. "Excuse me, you've dropped ... . Yes, look. It's passport." (something/ anything/ everything) 14. It's all finished. I am afraid there's ... left. (nothing/ anything/ something) 15. I heard a noise, but I didn't see ... . (any one/ no one) 16. It's too late. We can't do ... to help. (anything/ nothing) 17. I agree with

most of what he said, but I don't agree with ... . (something/ everything/ anything) 18. ... offered to help. They probably didn't have the time. (anybody/ nobody/ everybody) 19. If ... asks, you can tell them I'll be back soon. (somebody/ anybody/ everybody) 20. There was ... in the box, it was completely empty. (nothing/ anything/ something)

Exercise 16. Open the parentheses and give the correct form of the infinitive. 1. I am glad (read) this book now. 2. I hope (award) a scholarship for the coming semester. 3. He is happy (work) at this company for more than five years. 4. He does not like (interrupt) by anybody. 5. Ann was surprised (pass) the exams. 6. The question is too unexpected (answer) at once. 7. I want (solve) these equations. 8. This theorem was the first (prove). 9. She might (forget) to translate the text yesterday. 10. The question must (settle) an hour ago. 11. The article is (write) in time. 12. (Understand) the situation one must (know) the details.

Exercise 17. Complete the sentences by using infinitives. Supply a preposition after the infinitive if necessary.

Use the Model. 1. I'm planning to fly to the USA next year. 2. The student promised not … late for the lecture. 3. I need … my homework tonight. 4. I want … computer games after my classes. 5. He intends … a programmer when he graduates from the university. 6. I hope … all of my courses this term. So far my grades have been pretty good. 7. I try … class on time every day. 8. I learned (how) … when I entered the university. 9. I like … a lot of e-mails from my friends. 10. I hate … in front of a large group. 11. My roommate offered … me with my English.

Exercise 18. Write the correct form (gerund or infinitive) of the verbs given in parentheses. Sometimes more than one answer is possible.

1. He regrets (not study) harder when he was at school. 2. The teacher was very strict and nobody dared (talk) during his lessons. 3. She suggested (go) to the University by taxi. 4. (learn) English involves (speak) as much as you can. 5. (Solve) this equation multiply each term in it by the quantity that precedes it. 6. On (obtain) the data the scientists went on working. 7. The procedure (follow) depends

entirely on the student. 8. This equation must (solve) at the previous lesson. 9. Euclid was the first (bring) all the known facts about geometry into one whole system. 10. We don't mind (give) further assistance. 11. The method (apply) is rather complicated. 12. (prove) this theorem means (find) a solution for the whole problem. 13. Students are (study) the laws of mathematics and mechanics.

Exercise 19. Change the following passive sentences into active.

a. This frame of reference will be used to locate a point in space. b. Although solid analytic geometry was mentioned by R. Descartes, it was not elaborated thoroughly and exhaustively by him. c. Most uses of computers in language education can be described as CALL. d. Since many students are considerably more able as algebraists than as geometers, analytic geometry can be described as the "royal road" in geometry that Euclid thought did not exist. e. Now new technologies are being developed to make even better machines. f. Logarithm tables, calculus, and the basis for the modern slide rule were not invented during the twentieth century. g. After World War 2 ended, the transistor was developed by Bell Laboratories. h. The whole subject matter of analytic geometry was well advanced, beyond its elementary stages, by L. Euler.

Exercise 20. Read and translate the text.

History of Perfect Number

It is not known when Perfect Numbers were first discovered, or when they were studied, it is thought that they may even have been known to the Egyptians, and may have even been known before. Although the ancient mathematicians knew of the existence of Perfect Numbers, it was the Greeks who took a keen interest in them, especially Pythagoras and his followers (O'Connor and Robertson, 2004).

The Pythagoreans found the number 6 interesting (more for its mystical and numerological properties than for any mathematical significance), as it is the sum of its proper factors, i.e. $6 = 1 + 2 + 3$ This is the smallest Perfect Number, the next being 28 (Burton, 1980).

Though the Pythagoreans were interested in the occult properties of Perfect Numbers, they did little of mathematical significance with them. It was around 300 BC when Euclid wrote his Elements that the first real result was made. Although Euclid concentrated on Geometry, many number theory results can be found in his text (Burton, 1980).

We shall consider Euclid's result in a moment, but first, let's define Perfect Numbers more broadly. There are numerous ways to define Perfect Numbers, the early definitions being given in terms of aliquot parts. The author defines: A Perfect Number n, is a positive integer which is equal to the sum of its factors, excluding n itself.

Exercise 21. Discuss the following. What does the quotation mean? What is your interpretation or opinion of it? "Without mathematics, there's nothing you can do. Everything around you is mathematics. Everything around you is numbers." ~ Shakuntala Devi.

Exercise 22. Discuss the following. Math and other sciences.

Individual Work

Task 1. Over to you. Use the Internet to find a biography of a famous mathematician and his/her contribution to science. Make notes for a short presentation. In groups, try to agree on the three most important people in the history of math.

Task 2. Project Ideas Study the literature about famous scientists. Choose one scientist and find the following information about him: – Biography and Family; – Educational Background; – Scientific Achievements. Prepare a short report and a presentation on the achieved results. Share your ideas.

Task 3. Follow this link https://ed.ted.com/lessons/making-sense-of-irrational-numbers-ganesh-pai. Like many heroes of Greek myths, the philosopher Hippasus was rumored to have been mortally punished by the gods. But what was his crime? Did he murder guests or disrupt a sacred ritual? No, Hippasus's transgression was mathematically proving the hitherto unprovable. Ganesh Pai describes the history and math behind irrational numbers**.** Watch the video Making sense of irrational numbers as many times as you need. Make sure that you are throughall stages: ”Think”, “Dig Deeper” and “Discuss”. All necessary translations and thoughts you are welcome to take down into your copy-book.

Task 4. Follow this link

https://www.youtube.com/watch?v=64643Op6WJo . What is Mathematics? Explained using animations and illustration Video. Watch the video What is Mathematics? as many times as you need. Write down the main points and tell about it.

Task 5. Follow this link https://ed.ted.com/lessons/the-paradox-at-the-heart-of-mathematics-godel-s-incompleteness-theorem-marcus-du-sautoy?lesson_collection=math-in-real-life . Consider the following sentence: “This statement is false.” Is that true? If so, that would make the statement false. But if it’s false, then the statement is true. This sentence creates an unsolvable paradox; if it’s not true and it’s not false– what is it? This question led a logician to a discovery that would change mathematics forever. Marcus du Sautoy digs into Gödel’s Incompleteness Theorem. Watch the video The paradox at the heart of mathematics: Gödel's Incompleteness Theorem as many times as you need. Make sure that you are throughall stages: ”Think”, “Dig Deeper” and “Discuss”. All necessary translations and thoughts you are welcome to take down into your copy-book.

UNIT 10. IMAGINARY NUMBERS

Exercise 1. Read and memorise the following words and word combinations

Absorb into – включати в; поглинати

addition – додавання

basic unit – основна одиниця

binary system – бінарна; двійкова система

bear in mind – взяти до уваги; пам’ятати; мати на увазі

computation – обчислення, підрахунок, числення

correspond – відповідати; найбільш точно підходити

decimal positional system – десяткова позиційна система

decimal expansion – десяткове розширення; розкладання на десяткові дроби

decimal expression – десяткове вираження

exponential – показниковий; показникова функція

fraction – дріб; частина

hexadecimal system – шістнадцяткова система

multiplication - множення

octal system – вісімкова система

place-value system – позиційна система

reciprocal – еквівалентний; рівнозначний

tens - десятки units – одиниці

whole number – ціле число

‘over’ a whole number – ‘над’ цілим числом

bottom number – нижнє число

improper fraction – неправильний дріб

proper fraction – правильний дріб

left over – залишок

comprise – охоплювати; складатися з

common factor – спільний дільник

cancel the fraction – скоротити дріб

single stroke – одним ударом

adding powers – додавання степеней

count up – підраховувати; рахувати

even number – парне число

indivisible unit – неподільна одиниця

infinite number – нескінченне число

irrational number – ірраціональне число

fit flush with – збігатися з...

odd number – непарне число

Exercise 2. Write the transcription and translate the following words. Make sentences with them.

differential

derivative

variable

Leibniz notation

differential form

linear approximation

differential geometry

tangent vector

exterior derivative

differentiable function

linear part

partial derivative

differentiability

Pythagorean theorem

right triangle

hypotenuse

equation

Exercise 3. Read and translate the text

We can certainly imagine numbers. Sometimes I imagine my bank account is a million pounds in credit and there's no question that would be an 'imaginary number'. But the mathematical use of imaginary is nothing to do with this daydreaming. The label 'imaginary' is thought to be due to the philosopher and mathematician René Descartes, in recognition of curious solutions of equations which were definitely not ordinary numbers.

Do imaginary numbers exist or not? This was a question chewed over by philosophers as they focused on the word imaginary. For mathematicians the existence of imaginary numbers is not an issue.

They are as much a part of everyday life as the number 5 or π. Imaginary numbers may not help with your shopping trips, but go and ask any aircraft designer or electrical engineer and you will find they are vitally important. And by adding a real number and an imaginary number together we obtain what's called a 'complex number', which immediately sounds less philosophically troublesome. The theory of complex numbers turns on the square root of minus 1.

So what number, when squared, gives -1? If you take any non-zero number and multiply it by itself (square it) you always get a positive number. This is believable when squaring positive numbers but is it true if we square negative numbers? We can use -1×-1 as a test case. Even if we have forgotten the school rule that 'two negatives make a positive' we may remember that the answer is either -1 or $+1$. If we thought -1×-1 equalled -1 we could divide each side by -1 and end up with the conclusion that $-1 = 1$, which is nonsense. So we must conclude $-1 \times -1 = 1$, which is positive.

The same argument can be made for other negative numbers besides -1, and so, when any real number is squared the result can never be negative. This caused a sticking point in the early years of complex numbers in the 16th century. When this was overcome, the answer liberated mathematics from the shackles of ordinary numbers and opened up vast fields of inquiry undreamed of previously. The development of complex numbers is the ‘completion of the real numbers’ to a naturally more perfect system.

Exercise 4. Translate the following

Мистецтво прикрашати перемагає прагматизм технічних розрахунків: наука для науки стає вищою формою діяльності. По Платону, математика варварів – якого високого рівня розвитку досягла їх цивілізація – була лише мистецтвом, не звільненим від пут необхідності. Грецька філософія поєднала, таким чином, поняття, що належать до різних сфер – методичної та філософської. У трактатах з оптики та астрономії застосовувалися принципи геометрії, оскільки за допомогою дедуктивного методу можна було легко оминути все, що здавалося «наочним» та «практичним». Щоправда, залишається незрозумілим, як математики ставилися до такого визначення свого роду занять. Крім того, не слід переносити сучасне поняття «чистої» та «прикладної» математики на «нематеріальну» та «наочну» математику давніх, оскільки вони не збігаються. Говорячи про ідеал «безкорисливої» науки, не можна не торкнутися проблеми мотивації розвитку математики. Тут треба відрізняти явища, які відігравали роль зовнішніх чинників, від, які можна назвати внутрішніми. У першій групі слід виділити оптику та астрономію, які ми відносимо до фізики. А вчені давнини відносили до галузі математики. Сюди належить статика, вчення про рівновагу.

Exercise 5. Translate the following text from Ukrainian into English.

Філософи та математики

Що нам відомо про "внутрішню" мотивацію? Можна спробувати знайти її визначення в передмовах, якими математики, починаючи з Архімеда, випереджали свої твори. Виявляється, що «безкорисливі» дослідження не плід грецького стереотипу мислення. Вони припускають існування якогось співтовариства математиків, які дотримуються встановлених норм. Насамперед, ці вчені вважають за потрібне виправдовуватися в тому, що вони займаються наукою заради науки, їм це здається цілком природним. У разі вони лише уточнюють, чому вибрали саме математику, а чи не фізику чи теологію. Математика більш достовірна і строга, її предмет «постійніший», ніж фізика, і «доступніший», ніж теологія.

У Стародавній Греції математики становили свого роду «міжнародне» співтовариство, члени якого були розпорошені по всьому Середземномор'ю: у Греції, Малій Азії, Єгипті та на Сицилії. Вони підтримували особисті контакти та обмінювалися своїми роботами.

Насамперед, вчені прагнули передати колегам свої завдання, знайти рішення тих завдань, які надсилали їм, або критикувати невдалі рішення, запропоновані іншими. Так, деякі з них набували загальновизнаного авторитету: їм надсилали на відкликання наукові праці, вони, у свою чергу, розсилали їх самим, на їхню думку, гідним.

Попадалися серед них і самозванці, але викрити обман було легко: їм пропонували завдання, яке не має рішення, а вони запевняли, що вирішили його. Звичайно, такі контакти залишалися суто особистими, вони зовсім не схожі на відносини, що складаються між вченими в рамках сучасних інституцій. «Безкорислива» наука, таким чином, пов'язувалася з існуванням якоїсь групи, усередині якої панувала суперництво, що нагадує те, що відбувається серед сучасних учених. При чому, таке порівняння не цілком правомірне, надто вже відчутна різниця масштабів цих спільнот: в епоху еллінізму кількість вчених, особливо математиків, не перевищувала кількох сотень.

Під час римського панування найкращі автори (Птолемей, Папп) займалися вже лише уточненням отриманих результатів. Суперництво і пошук нового пішли в минуле разом з епохою, що породила їх.

Exercise 6. Match the terms from the left column and the definitions from the right column:

negative	designating a number or a quantity expressible as a quotient of two integers, one of which may be unity

positive	a set of numbers or other algebraic elements for which arithmetic operations (except for division by zero) are defined in a consistent manner to yield another element of a set
rational	designating a quantity greater than zero or one to be added
irrational	the number of elements in a given group
order	designating a real number not expressible as an integer or as a quotient of two integers
quotient	a mathematical set containing some or all of the elements of a given set
subset	a quantity less than zero or one to be subtracted
field	any positive or negative number or zero: distinguished from fraction
order	the result obtained when one number is divided by another number

Exercise 7. Give the English equivalents of the following words and word combinations:

Граничне значення, кінцева послідовність, область визначення, підсумовування, розбіжний ряд, нескінченна послідовність, кінцевий ряд, нескінченний ряд, що сходиться послідовність, верхня межа, загальний член, нижня межа, невід'ємне ціле число.

Exercise 8. Form nouns from the following adjectives by adding suffix “ness”. Translate them into Ukrainian.

Example: careful – carefulness; careless – carelessness

1. abrupt 2. concise 3. homeless 4. colourful 5. aware 6. polite 7. thoughtful 8. compact 9. conscious 10. good 11. fruitful 12. fruitless 13. bright 14. expensive 15. thick.

Exercise 9. Translate the following words and word-combinations into English:

1. ієрархія чисел 2. рамки 3. легкість у обробці 4. округлені помилки 5. векторні числення 6. теорія хаосу 7. числення по множині 8. демонструвати непередбачену але детерміністську поведінку 9. робити точним 10. обмеження 11. такий, що не може бути виконаний 12. тензорне обчислення 13. ціле число 14. підмножина раціональних чисел 15. розподіл на гілки.

Exercise 10. Match the terms from the left column and definitions from the right column:

calculus	a fixed quantity or value which a varying quantity is regarded as approaching indefinitely
differential calculus	the rate of continuous change in variable quantities
integral calculus	the point in a body, or in a system of bodies, at which, for certain purposes, the entire mass may be assumed to be concentrated
limit	the branch of mathematics dealing with derivatives and their applications
volume	having the three dimensions of length, breadth and thickness (prisms and other solid figures)
centroid	a part of a figure, esp. of a circle or sphere, marked off or made separate by a line or plane, as a part of a circular area bounded by an arc and its chord, b) any of a finite sections of a line
curvee	the path of a moving point, thought of as having length but not breadth, whether straight or curved

solid	the combined methods of mathematical analysis of differential and integral calculus
line	the limiting value of a rate of change of a function with respect to variable; the instantaneous rate of change, or slope, of a function
segment	the sum of a sequence, often infinite, of terms usually separated by plus or minus signs
derivative	the slope of a tangent line to a given curve at a designated point
fluxion	the branch of higher mathematics that deals with integration and its use in finding volumes, areas, equations of curves, solutions of differential equations
slope	a one-dimensional continuum of in a space of two or more dimensions
series	any system of calculation using special symbolic notation
infinitesimal calculus	the amount of space occupied in three dimensions; cubic contents or cubic magnitude

Exercise 11. Give the English equivalents of the following words and word combinations:

Рівносторонній трикутник, правильний трикутник, замкнута плоска фігура, апофема (радіус вписаного кола), вписане коло, внутрішній кут, взаємно прості, радіус описаного кола, опуклий багатокутник, описане коло, пентаграма (п'ятикутна зірка) багатокутник, багатокутник у вигляді зірки, правильний багатокутник у вигляді зірки, зовнішній кут, перпендикуляр, рівносторонній багатокутник.

Exercise 12. Read and translate the text

Deduction and Induction

The scientists have proved a chain of theorems and have come to recognize the entire structure of undefined terms, definitions, assumptions, and theorems as constituting an abstract logical system. In such a system we say that each proposition is derived from its predecessor by the process of logical deduction. This process of logical deduction is scientific reasoning.

This scientific reasoning must not be confused with the mode of thinking employed by the scientist when he is feeling his way toward a new discovery. At such times the scientist, curious about the sum of the angles of a triangle, proceeds to measure the angles of a great many triangles very carefully. In every instance he notices that the sum of the three angles is very close to 180°; so he puts forward a guess that this will be true of every triangle he might draw.

This method of deriving a general principle from a limited number of special instances is called induction. The method of induction always leaves the possibility that further measurement and experimentation may necessitate some modification of the general principle.

The method of deduction is not subject to upsets of this sort. When the mathematician is groping for (шукає) new mathematical ideas, he uses induction. On the other hand, when he wishes to link his ideas together into a logical system, he uses deduction. The laboratory scientist also uses deduction when he wishes to order and classify the results of his observations and his inspired guesses and to arrange them all in a logical system. While building this logical system he must have a pattern (модель) to guide him, an ideal of what a logical system ought to be.

The simplest exposition (виклад) of this ideal is to be found in the abstract logical system of demonstrative geometry. It is clear that both deductive and inductive thinking are very useful to the scientist.

Exercise 13. Ask special questions to which the sentences below are the answers.

1. A statement satisfying certain conditions is true. (what) 2. Like terms being arranged in the following way will be enclosed in the parenthesis. (where) 3.

Reference is made to the commonly accepted system. (what ... to) 4. The force keeping all material bodies including people on the Earth is called gravitation. (what kind) 5. Having used the classification suggested by my science adviser I found it very convenient. (when) 6. Having been given little information they couldn't continue the research. (why) 7. Having followed the procedure they obtained the required results. (how) 8. Any fraction represents the quotient of its numerator divided by its denominator. (what) 9. Having obtained a proper interpretation of this fact they realized the importance of the problem. (when) 10. The created method has no advantages over the old one. (what) 11. Differential equation is an equation containing differentials or derivatives of a function of one independent variable. (what)

Exercise 14. Use the questions in indirect speech following the model (Sequence of Tenses).

Model: 1. They said, "We will go to the South". They said that they would go to the South.

2 She said, "I have done the test." She said that she had done the test. 3. They said, "We saw the film two years ago." They said that they had seen the film two years before." 4. He said, "I am taking driving lessons." He said that he was taking driving lessons. 5. She said, "I was reading for the exam at that time." She said she had been reading for the exam at that time. 1. He said, "I am working on my diploma paper project." 2. She said, "I haven't been to the lecture." 3. They said, "We won't come to the party." 4. She said, "We have installed a new antivirus program". 5. He said, "I wrote the article three years ago." 6. They said, "We won't go to France." 7. He said, "I was working at five o'clock." 8. She said, "I have been waiting for you since three o'clock." 9. They said, "The lecture will be held in the assembly hall."

Exercise 15. Choose the correct form of the Participle.

1. (to name) geometric ideas we usually use letters of the alphabet. 2. We insisted on the (to follow) notation of the geometric object. 3. (to divide) both the numerator and the denominator by x you will get the following expression. 4.

When (to speak) with my science adviser I got better understanding of the latest development in my special field. 5. The properties of the material (to use) in the experiment now are given in the latest article. 6. The advantages of the new system (to prove) by many tests are very important. 7. Two angles (to have) the same vertex and a common side are refered to as adjacent angles. 8. The concepts (to introduce) at the seminar should be considered in detail. 9. The (to obtain) difference must be checked carefully. 10. The (to expect) result must prove that this law holds for similar cases.

Exercise 16. The Continuous or Perfect Continuous Tenses.

1. I (to look for) a photographs my brother sent to me. 2. They (to have) a meeting now. 3. The phone always (to ring) when I (to have) a bath. 4. Friends always (to talk) to me when I (to try) to concentrate. 5. He (to watch) television when the door bell (to ring). 6. He (to build up) his business all his life. 7. They (to stay) with us for a couple of weeks. 8. By 1992 he (to live) there for ten years. 9. The video industry (to develop) rapidly. 10. He (to work) nights next week. 11. She (to spend) this summer in Europe. 12. Why are you so late? I (to wait) you for hours. 13. The boys must be tired. They (to play) football in the garden all afternoon. 14. The old town theatre is currently (to rebuild). 15. I usually (to go) to work by car, but I (to go) on the bus this week while my car (to repair).

Exercise 17. Perfect Tenses. Complete the sentences using the following words: already before ever for just by since so still yet never

1. Have you … dreamt of going to London? 2. I haven't worked out how to set the timer on the video …. 3. My dad's lived in the same house … he was born. 4. The film's only been on … a couple of minutes. 5. Kate has passed three exams out of five … far. 6. He will have translated the text … 3 o'clock tomorrow. 7. He's only … got home. 8. It's eleven o'clock and he … hasn't come home. Where could he be? 9. I've … met Ann …. What's she like? 10. He has … finished doing his homework.

Exercise 18. Transform the sentences from Perfect Active into Perfect Passive.

1. She has just typed her report for the conference. 2. The teacher told us that she had checked all the tests. 3. The student will have written his degree work by May. 4. They have learnt a lot of new English words. 5. He hasn't found the answer yet. 6. I've just received my exam results. 7. By the end of the conference, the participants had discussed a number of important questions concerning the problem. 8. They will have read two books on topology by the end of the month. 9. We had planned the meeting months in advance, but we still had problems. 10. I had discussed the plan of my work with my science adviser before the end of the class.

Exercise 19. Ask special questions using question words given in parentheses.

The development of geometry

1. The earliest recorded beginnings of geometry can be traced to early predecessors. (to whom) 2. They discovered obtuse triangles in the ancient Indus Valley and ancient Babylonia from around 3000 BC. (where; when) 3. Early geometry was a collection of empirically discovered principles concerning lengths, angles, areas, and volumes. (what collection) 4. In geometry a spatial point is a primitive notion upon which other concepts may be defined. (where) 5. Points have neither volume, area, length, nor any other higher dimensional analogue. (what (question to the subject)) 6. In branches of mathematics dealing with a set theory, an element is often referred to as a point. (where; how) 7. A point could also be defined as a sphere which has a diameter of zero. (how).

Exercise 20. Join the two sentences to make one sentence, beginning with a gerund. Model. She's a teacher. It's hard work. Being a teacher is hard work / Teaching is hard work. 1. Capital letters are used to name geometrical objects. It is very convenient. 2. You are to classify these quadrilaterals. It requires the knowledge of some properties. 3. We are going to locate this point on the y axis. It will give us the first point on the line. 4. The student intends to divide a circle into a certain number of congruent parts. It will help him to obtain a regular polygon. 5. The base and the altitude of a rectangle are to be multiplied. It will give the product of its dimensions or the area of the rectangle. 6. Don't argue! It's no use. In a crossed quadrilateral, the interior angles on either side of the crossing add up to 720°. 7. Don't deny this fact! It is useless. A square is a quadrilateral, a parallelogram, a rectangle and a rhombus. 8. You are going to divide a heptagon (a 7-sided polygon) into five triangles. Is it any good?

Exercise 21. Complete the sentences using a gerund as an attribute.

1.I didn't very much like the idea of … . 2. What is the purpose of … ? 3. She had no difficulty (in) … . 4. You have made great progress in … . 5. He was late, and he was afraid of … . 6. Can you imagine the pleasure of … . 7. He always produces the impression of … . 8. I am afraid you do not realize the importance of … .

Exercise 22. Complete the second sentence so that it has a similar meaning to the first one. Use the word in bold and other words to complete each sentence.

1. I'll be happy when I can have a rest after exams. forward to I'm looking ... a rest after exams. 2. Learning new geometric theorems is something I like doing. interested in I'm always ... new geometric theorems 3. If I study a lot at night, it keeps me awake. prevents from ... a lot at night ... sleeping. 4. I often operate the computer at university. am used to I ... the computer at university. 5. He didn't want to take the books back to the library. feel like He didn't... the books back to the library. 6. He hates it if he has to do a lot of boring exercises. can't stand He ... a lot of boring exercises. 7. 'I'm sorry. I've broken the speed limit', said Sue. apologized for Sue ... the speed limit. 8. Let us write a new program. suggest I ... a new program.

Exercise 23. Read and translate the text

An imaginary number is a number that, when squared, has a negative result. Essentially, an imaginary number is the square root of a negative number and does not have a tangible value. While it is not a real number — that is, it cannot be quantified on the number line — imaginary numbers are "real" in the sense that they exist and are used in math.

Imaginary numbers, also called complex numbers, are used in real-life applications, such as electricity, as well as quadratic equations. In quadratic planes, imaginary numbers show up in equations that don’t touch the x axis. Imaginary numbers become particularly useful in advanced calculus.

Usually denoted by the symbol *i*, imaginary numbers are denoted by the symbol *j* in electronics (because *i* already denotes "current"). Imaginary numbers are particularly applicable in electricity, specifically alternating current (AC) electronics. AC electricity changes between positive and negative in a sine wave. Combining AC currents can be very difficult because they may not match properly on the waves. Using imaginary currents and real numbers helps those working with AC electricity do the calculations and avoid electrocution.

Imaginary numbers can also be applied to signal processing, which is useful in cellular technology and wireless technologies, as well as radar and even biology (brain waves). Essentially, if what is being measured relies on a sine or cosine wave, the imaginary number is used.

IMAGINARY NUMBERS CHART

There is also an interesting property of *i*. When you multiply it, it cycles through four different values. For example, $i \times i = -1$. Then, $-1 \times i = -i$. $-i \times i = 1$. Then $1 \times i = i$, coming full circle. This makes exponents of *i* easy to figure out. If:

$i = \sqrt{-1}$ $\quad i^2 = -1$ $\quad i^3 = -\sqrt{-1}$ $\quad i^4 = 1$ $\quad i^5 = \sqrt{-1}$

This cycle will continue through the exponents, also known as the imaginary numbers chart. Knowledge of the exponential qualities of imaginary numbers is useful in the multiplication and division of imaginary numbers. After grouping the coefficients and the imaginary terms, the rules of exponents can be applied to *i* while the real numbers are multiplied as normal. The same is done with division. By applying the usual multiplication and division rules, imaginary numbers can be simplified as you would with variables and coefficients.

■**Exercise 24. Discuss the following.** What does the quotation mean? What is your interpretation or opinion of it? "The intelligence of the creature known as a crowd, is the square root of the number of people in it." ~ Terry Pratchett

Individual Work

Task 1. Over to you. Work in pairs, A and B. Roleplay the situation. Student A You are an interviewer. Come up with some questions that help to reveal information or story about your partner, who is a talented mathematician. Take short notes. Student B You are a famous mathematician. Get ready to answer questions about your life and career.

Task 2. Project Ideas Study the literature about Sieve Theories. Choose one of the sieve theories, prepare a report and a presentation on the achieved results. Share your ideas.

Task 3. Follow this link https://ed.ted.com/lessons/your-brain-can-solve-algorithms-david-j-malan. An algorithm is a method of solving problems both big and small. Though computers run algorithms constantly, humans can also solve problems with algorithms. David J. Malan explains how algorithms can be used in seemingly simple situations and also complex ones. Watch the video What's an algorithm? as many times as you need. Make sure that you are throughall stages: ”Think”, “Dig Deeper” and “Discuss”. All necessary translations and thoughts you are welcome to take down into your copy-book.

Task 4. Follow this link **https://www.youtube.com/watch?v=N7RbmRma7A4** When going through your day, you may not realize how much math surrounds you. Second grader, Jim Patrick, sees math everywhere. From the fractions in the pizza you eat for lunch to the multiplication you can use to check the time, Jim encourages us all to recognize the math in our lives. Watch the video Math is everywhere as many times as you need. Write down the main points and tell about it.

Task 5. Follow this link https://ed.ted.com/lessons/the-mathematics-of-history-jean-baptiste-michel. What can mathematics say about history? According to TED Fellow Jean-Baptiste Michel, quite a lot. From changes to language to the deadliness of wars, he shows how digitized history is just starting to reveal deep underlying patterns. Watch the video The mathematics of history as many times as you need. Make sure that you are throughall stages: ”Think”, “Dig Deeper” and “Discuss”. All necessary translations and thoughts you are welcome to take down into your copy-book.

READING

Text 1. FERMAT'S LAST THEOREM

Task 1. Read the text about Fermat'slast theorem

Pierre de Fermat was born in Toulouse in 1601 and died in 1665. Today we think of Fermat as a number theorist, infact as perhaps the most famous number theorist who ever lived. The history of Pythagorean triples goes back to 1600 B.C, but it was not until the seventeenth century A.D that mathematicians seriously attacked, in general terms, the problem of finding positive integer solutions to the equation x n + yn = zn. Many mathematicians conjectured that there are no positive integer solutions to this equation if n isgreater than 2. Fermat's now famous conjecture was inscribed in the marginof his copy of the Latin translation of

Diophantus's Arithmetica. The note read: "To divide a cube intotwo cubes, a fourth power or in general any power whatever into two powers of the same denomination above the second is impossible and I have assuredly found an admirable proof of this, but the margin is too narrow to contain it".

Despite Fermat's confident proclamation the conjecture, referred to as "Fermat's last theorem" remains unproven. Fermat gave elsewhere a proof for the case n = 4. it was not until the next century that L. Euler supplied a proof for the case n = 3, and still another century passed before A. Legendre and L. Dirichlet arrived at independent proofs of the case n = 5. Not long after, in 1838, G. Lame established the theorem for n = 7. In 1843, the German mathematician E. Kummer submitted a proof of Fermat's theorem to Dirichlet.

Dirichlet found an error in the argument and Kummer returned to the problem. After developing the algebraic "theory of ideals", Kummer produced a proof for "most small n". Subsequent progress in the problem utilized Kummer's ideals and many more special cases were proved. It is now known that Fermat’s conjecture is true for all $n < 4.003$ and many special values of n, but no general proof has been found. Fermat’s conjecture generated such interest among mathematicians that in 1908 the German mathematician P. Wolfskehl bequeathed DM 100.000 to the Academy of Science at Gottingen as a prize for the first complete proof of the theorem. This prize induced thousands of amateurs to prepare solutions, with the result that Fermat’s theorem is reputed to be the maths problem for which the greatest number of incorrect proofs was published. However, these faulty arguments did not tarnish the reputation of the genius – P. Fermat.

Richard Lawrence Taylor (born 19 May 1962) is a British mathematician working in the field of number theory. A former research student of Andrew Wiles, he returned to Princeton to help his advisor complete the proof of Fermat's Last Theorem. Taylor received a $3 million 2014 Breakthrough Prize in Mathematics "For numerous breakthrough results in the theory of automorphic forms, including the Taniyama-Weil conjecture, the local Langlands conjecture for general linear groups, and the Sato-Tate conjecture." He also received the 2007 Shaw Prize in Mathematical Sciences for his work on the Langlands program with Robert Langlands.

Task 2. Answer the following questions.

1.How old was Pierre Fermat when he died?

2.Which problem did mathematicians face in the 17 century A.D?

3.What did many mathematiciansconjecture at that time?

4.Who first gave a proof to Fermat’s theorem?

5.What proof did he give?

6.Did any mathematicians prove Fermat’s theorem after him?

7.Who were they?

Task 3. Are the statements True (T) or False (F)? Correct the false sentences.

The German mathematician E. Kummer was the first to find an error in the argument.

With the algebraic “theory of ideals” in hand, Kummer produced a proof for “most small n” and many special cases.

A general proof has been found for all value of n.

The German mathematician P. Wolfskehl won DM 100.000 in 1908 for the first complete proof of the theorem.

Task 4. Translate into Ukrainian.

1. Fermat’s theorem The theorem that if a is an integer and p is a prime that does not divide a, then p does divide 1 1 p a − − ; or in congruence notation, 1 1 (mod) p a p − ≡ . For example, 84 – 1 is divisible by 5. A simple corollary is that, whether p divides a or not, it must divide p a a − : equivalently (mod) p aa p ≡ .

2. Fermat’s last theorem The conjecture that if the integer n is at least 3 then there are no integers x, y, z, none of which is zero, satisfying: nnn x + y z = Work on Fermat’s last theorem has provided much stimulus to the development of algebraic number theory; the impossibility of finding non zero integers x, y, z to satisfy the given equation has now been established for every n between 3 and 125000 inclusive.

Text 2. J. E. FREUND’S SYSTEM OF NATURAL NUMBERS POSTULATES

Task 1. Read the text to get more information about J. E. Freund’s System of Natural Numbers Postulates.

Modern mathematicians are accustomed to derive properties of natural numbers from a set of axioms or postulates (i.e., undefined and unproven statements that disclose the meaning of the abstract concepts). The well known system of 5 axioms of the Italian mathematician, Peano provides the description of natural numbers.

These axioms are: First: 1 is a natural number. Second: Any number which is a successor (follower) of a natural number is itself a natural number. Third: No two natural numbers have the same follower. Fourth: The natural number 1 is not the follower of any other natural number. Fifth: If a series of natural numbers includes both the number 1 and the follower of every natural number, then the series contains all natural numbers. The fifth axiom is the principle (law) of math induction.

From the axioms it follows that there must be infinitely many natural numbers since the series cannot stop. It cannot circle back to its starting point either because 1 is not the immediate follower of any natural number. In essence, Peano's theory states that the series of natural numbers is well ordered and presents a general problem of quantification. It places the natural numbers in an ordinal relation and the commonest example of ordination is the counting of things.

The domain of applications of Peano's theory is much wider than the series of natural numbers alone e.g., the relational fractions 111 1, 234 and so on, satisfy the axioms similarly. From Peano's five ruleswe can state and enumerate all the familiar characteristics and properties of natural numbers. Other mathematicians define these properties in terms of 8 or even 12 axioms (J. E. Freund) and these systems characterize properties of natural numbers much more comprehensively and they specify the notion of operations both arithmetical and logical.

Note that sums and products of natural numbers are written as a + b and a. b or ab, respectively. Postulate No.1: For every pair of natural numbers, a and b, in that order, there is a unique (one and only one) natural number called the sum of a and b. Postulate No.2: If a and b are natural numbers, then a + b = b + a Postulate No.3: If a, b and c are natural numbers, then (a + b) + c = a + (b + c) Postulate No.4: For every pair of natural numbers, a and b, in that order, there is a unique (one and only one) natural number called the product. Postulate No.5: If a and b are natural numbers, then ab = ba Postulate No.6: If a, b and c are natural numbers, then (ab)c = a(bc) Postulate No.7: If a, b and c are natural numbers, then a(b + c) = ab + ac Postulate No.8: There is a natural number called "one" and written 1 so that if a is an arbitrary natural number, then a.1 = a Postulate No.9: If a, b and c are natural numbers and if ac = bc then a = b Postulate No.10: If a, b and c are natural numbers and if a + c = b + c then a = b Postulate No.11: Any set of natural numbers which (1) includes the number 1 and which (2) includes a + 1 whenever it includes the natural number a, includes every natural number. Postulate No.12: For any pair of natural numbers, a and b, one and only one of the following alternatives must hold: either a = b, or there is a natural number x such that a + x = b, or there is a natural number y such that b + y = a. Freund's system of 12 postulates provides the possibility to characterize natural numbers when we explain how they behave and what math rules they must obey. To conclude the definition of "natural numbers" we can say that they must be interpreted either as standing for the whole number or else for math objects which share all their math properties. Using these postulates mathematicians are able to prove all other rules about natural numbers with which people have long been familiar.

Task 2. Answer the questions.

1. How many axioms did the Italian mathematician Peano give? What were they?

2. Which axiom is the most important? Why?

3. What does Peano's theory state in essence?

4. What can we state from Peano's five rules?

5. Who developed these axioms? What did he do?

6. How useful is Freund's system of 12 postulates?

Task 3. Work in pairs

1. Complete the formulae written by Freund's system of 12 postulates. If a, b, c are natural numbers:

a + b =............................

a (b + c) =...............................

(a + b) + c =............................

a.1 =................................

ab =............................

$ac = bc \Rightarrow$ ....

$(ab)c =$............................

$a + c = b + c \Rightarrow$ ....

2. Practice speaking them based on the 12 postulates.

Work in pairs

Look at these symbols

$a \in S$; $b \notin S$

What do they mean?

3.Try to fill in the gaps with the words you hear.

In all (1)………………….. of mathematics, we are concerned with collections of objects of one kind or another. In basic algebra (2) ………………….were the principle objects of investigation. The terms (3)……………………. and (4)……………….…… are undefined but are taken to be (5)…………………….. . A description, or a property of the objects which (6)……………………. to a set must be clearly stated. We note that an object which belongs to a set may itself be a set. If an object belongs to a set, it is called a (7)………………... or (8)…………… of that set. The symbol a S ∈ means that a is an element of the set S. It is customary, in elementary set theory, to denote sets by (9)………….. and elements of sets by (10)…………………….. . Also b S ∉ signifies that b is not an element of S.

Task 4. Translate into Ukrainian

1.A set A of real numbers is said to be inductive if, and only if, 1 A∈ and x∈A implies (1) x + ∈A. 2. The real number system must have any property which is possessed by a field, an ordered field, or a complete ordered field. 3. A real number is called a rational number if, and only if, it is the quotient of two integers. A real number which is not rational is said to be irrational.

Text 3. The Role of Mathematics and Mathematical Analysis.

Task 1. Read the text about the Role of Mathematics and Mathematical Analysis.

In mathematics and, in particular, in mathematical analysis practical work and observation of nature are, as in other sciences, the main source of scientific discoveries. In their turn, mathematical methods play a very important role in natural sciences and engineering.

Mathematical methods lie in the foundation of physics, mechanics, engineering and other natural sciences. For all of them mathematics is a powerful theoretical and practical tool without which no scientific calculation and no engineering and technology are possible.

Mathematical analysis which treats of variables and functional relationships between them is particularly important since the laws of physics, mechanics, chemistry, etc. are expressed as such relationships. An important feature of the application of mathematics to other sciences is that in enables us to make scientific pre dictions, that is to draw, on the basis of logic and with the aid of mathematical methods, correct conclusions whose agreement with reality is then confirmed by experience, experiment and practice. Here is one remarkable example illustrating what has been said.

As it is known, the modern science of aviation was created by the famous scientist Professor M. Zhukovsky (1847-1921). He derived by means of mathematical methods certain formulas and laws which enabled him to predict the possibility of aerobatics, and, in particular, of looping the loop. In recent years the role of mathematics has still increased especially in connection with the appearance of modern high-speed electronic computers.

Realization of space flights, launching rockets to other planets and establishing radio and television communication with them require extremely complicated and precise mathematical calculations which cannot be performed without computers. Mathematical methods are penetrating deeply even into such traditionally “nonmathematical” sciences as economics, biology, medicine, etc. It can be said that no modern scientific and technical project can be realized without mathematics and its methods.

Task 2. Answer the following questions:

1. What is the role of mathematical methods?
2. What does mathematical analysis consider?
3. Who created modern aviation science?
4. What did Zhukovsky deduce with the help of mathematical methods?
5. What is the role of mathematics in recent years?

Task 3. Match the numbers to the letters:

1. theoretical and practical tool	a) наукові передбачення
2. variables and functional relationships	b) наука про число і простір
3. science of number and space	c) застосування математики
4. scientific pre dictions	d) теоретичний та практичний інструмент
5. application of mathematics	e) змінні та функціональні взаємозв'язки

Text 4. Mathematics as a subject

Task 1. Read the text about mathematics as a subject. Be ready to discuss.

There are many universities and institutes of education in our country. Every year hundreds of graduates get jobs at schools and boarding-schools. Soon I'll graduate from the Institute and get a teacher's certificate. My dream to be a teacher of mathematics will come true.

Mathematics is the science of number and space, of all relations, of structure in the broadest sense. I got interested in mathematics at school. This science has always attracted me by its harmony, exactness, laconism of statements, logic and by beauty of mathematical solutions.

My school teacher of mathematics taught us love of this subject, to display its beauty. Thus, on leaving school I didn't hesitate about the choice of my future profession. I firmly decided to study mathematics.

Now I am a third-year student. The course for teachers of mathematics lasts four years. We study mathematics, history, pedagogical science, psychology, mathematics teaching methods and other subjects. The study of a foreign language is also compulsory. My favourite subjects are algebra, geometry, mathematical analysis and mathematical logic. The chief method of teaching in all the subjects is the lecture method. Of special attention are teaching practice at school and the students independent work. While at teaching practice at school I realized that it wasn't enough to be a good mathematician and know this subject well.

The teacher must arouse and maintain the interest in his pupils, work out and apply new methods of teaching. Mathematical subjects are paid much attention in the system of school instruction. In the recent years the teaching level of mathematics at school has been raised. That is why teachers must again and again improve their knowledge and teaching methods. Soon I'll get a job in a school. I'll teach my pupils love of mathematics and make it significant and enjoyable for many more of them. I'll do my best to help them understand the ways of people and widen their knowledge and ideas of the world. I want my pupils to be honest, truthful, hardworking and eager for knowledge. If they love mathematics as much as I do, I'll be the happiest person on earth.

Task 2. Work in pairs. Discuss. Use these questions to help you.

1. What do you know about mathematics as a science?
2. What scientists do you know that have studied of mathematics?
3. What is the science of mathematics?
4. What famous mathematicians do you know?
5. When did you get interested in mathematics?
6. When did you decide to become a teacher of mathematics?
7. Who taught you love of mathematics?
8. Are lectures on mathematics teaching methods interesting?

9. Do you think will make a good teacher of mathematics?

10. When did the problems of school mathematics teaching methods become more interesting to you?

11. Do mathematical subjects really play a very important part in the system of school instruction?

Task 3. Translate into Ukrainian and make the report on Cavalieri's principle to the group.

In geometry, Cavalieri's principle, sometimes called the method of indivisibles, named after Bonaventura Cavalieri, is as follows:

2-dimensional case: Suppose two regions in a plane are included between two parallel lines in that plane. If every line parallel to these two lines intersects both regions in line segments (отрезок прямой, линейный сегмент) of equal length, then the two regions have equal areas.

3-dimensional case: Suppose two regions in three-space (solids) are included between two parallel planes. If every plane parallel to these two planes intersects both regions in cross-sections of equal area, then the two regions have equal volumes.

Today Cavalieri's principle is seen as an early step towards integral calculus, and while it is used in some forms, such as its generalization in Fubini's theorem, results using Cavalieri's principle can often be shown more directly via integration. In the other direction, Cavalieri's principle grew out of the ancient Greek method of exhaustion, which used limits but did not use infinitesimals.

Text 5. Main branches of Mathematical Analysis

Task 1. Read the text about main branches of mathematical analysis.

Real analysis. Real analysis (traditionally, the theory of functions of a real variable) is a branch of mathematical analysis dealing with the real numbers and realvalued functions of a real variable. In particular, it deals with the analytic properties of real functions and sequences, including convergence and limits of sequences of real numbers, the calculus of the real numbers, and continuity, smoothness and related properties of real-valued functions.

Complex analysis, traditionally known as the theory of functions of a complex variable, is the branch of mathematical analysis that investigates functions of complex numbers. It is useful in many branches of mathematics, including algebraic geometry, number theory, applied mathematics; as well as in physics, including hydrodynamics, thermodynamics, mechanical engineering, electrical engineering, and particularly, quantum field theory.

Complex analysis is particularly concerned with the analytic functions of complex variables (or, more generally, meromorphic functions). Because the separate real and imaginary parts of any analytic function must satisfy Laplace's equation, complex analysis is widely applicable to two-dimensional problems in physics.

Functional analysis. Functional analysis is a branch of mathematical analysis, the core of which is formed by the study of vector spaces endowed with some kind of limit-related structure (e.g. inner product, norm, topology, etc.) and the linear operators acting upon these spaces and respecting these structures in a suitable sense. The historical roots of functional analysis lie in the study of spaces of functions and the formulation of properties of transformations of functions such as the Fourier transform as transformations defining continuous, unitary etc. operators between function spaces. This point of view turned out to be particularly useful for the study of differential and integral equations.

Differential equations. A differential equation is a mathematical equation for an unknown function of one or several variables that relates the values of the function itself and its derivatives of various orders. Differential equations play a prominent role in engineering, physics, economics, biology, and other disciplines.

Differential equations arise in many areas of science and technology, specifically whenever a deterministic relation involving some continuously varying quantities (modeled by functions) and their rates of change in space and/or time (expressed as derivatives) is known or postulated. This is illustrated in classical mechanics, where the motion of a body is described by its position and velocity as the time value varies. Newton's laws allow one (given the position, velocity, acceleration and various forces acting on the body) to express these variables dynamically as a differential equation for the unknown position of the body as a function of time. In some cases, this differential equation (called an equation of motion) may be solved explicitly.

Measure theory. A measure on a set is a systematic way to assign a number to each suitable subset of that set, intuitively interpreted as its size. In this sense, a measure is a generalization of the concepts of length, area, and volume.

A particularly important example is the Lebesgue measure on a Euclidean space, which assigns the conventional length, area, and volume of Euclidean geometry to suitable subsets of the -dimensional Euclidean space. For instance, the Lebesgue measure of the interval in the real numbers is its length in the everyday sense of the word – specifically, 1. Technically, a measure is a function that assigns a non-negative real number or +∞ to (certain) subsets of a set. It must assign 0 to the empty set and be (countably) additive: the measure of a 'large' subset that can be decomposed into a finite (or countable) number of 'smaller' disjoint subsets, is the sum of the measures of the "smaller" subsets.

In general, if one wants to associate a consistent size to each subset of a given set while satisfying the other axioms of a measure, one only finds trivial examples like the counting measure. This problem was resolved by defining measure only on a sub-collection of all subsets; the so-called measurable subsets, which are required to form a -algebra. This means that countable unions, countable intersections and complements of measurable subsets are measurable. Nonmeasurable sets in a Euclidean space, on which the Lebesgue measure cannot be defined consistently, are necessarily complicated in the sense of being badly mixed up with their complement. Indeed, their existence is a non-trivial consequence of the axiom of choice.

Numerical analysis. Numerical analysis is the study of algorithms that use numerical approximation (as opposed to general symbolic manipulations) for the problems of mathematical analysis (as distinguished from discrete mathematics).

Modern numerical analysis does not seek exact answers, because exact answers are often impossible to obtain in practice. Instead, much of numerical analysis is concerned with obtaining approximate solutions while maintaining reasonable bounds on errors. Numerical analysis naturally finds applications in all fields of engineering and the physical sciences, but in the 21st century, the life sciences and even the arts have adopted elements of scientific computations. Ordinary differential equations appear in celestial mechanics (planets, stars and galaxies); numerical linear algebra is important for data analysis; stochastic differential equations and Markov chains are essential in simulating living cells for medicine and biology.

Other topics in mathematical analysis:

- Calculus of variations deals with extremizing functionals, as opposed to ordinary calculus which deals with functions.
- Harmonic analysis deals with Fourier series and their abstractions.
- Geometric analysis involves the use of geometrical methods in the study of partial differential equations and the application of the theory of partial differential equations to geometry.
- Clifford analysis, the study of Clifford valued functions that are annihilated by Dirac or Dirac-like operators, termed in general as monogenic or Clifford analytic functions p-adic analysis, the study of analysis within the context of p-adic numbers, which differs in some interesting and surprising ways from its real and complex counterparts.
- Non-standard analysis, which investigates the hyperreal numbers and their functions and gives a rigorous treatment of infinitesimals and infinitely large numbers.
- Computable analysis, the study of which parts of analysis can be carried out in a computable manner.

 Stochastic calculus – analytical notions developed for stochastic processes.

 Set-valued analysis – applies ideas from analysis and topology to set-valued functions.

 Convex analysis, the study of convex sets and functions. Techniques from analysis are also found in other areas such as: physical sciences.

The vast majority of classical mechanics, relativity, and quantum mechanics is based on applied analysis, and differential equations in particular. Examples of important differential equations include Newton's second law and the Einstein field equations. Functional analysis is also a major factor in quantum mechanics.

Task 2. Answer the questions.

1. What mathematical notions does the Real analysis deal with?

2. What types of functions does the Complex analysis concerned with?

3. Describe the historical roots of functional analysis.

5. What kind of disciplines do the differential equations play a prominent role in?

6. Referring to the measure theory how can the measure of a 'large' subset be decomposed into?

7. What fields does the Numerical analysis find its applications in?

8. Enumerate the basic forms of Mathematical Analyses and expand on their principles.

Task 3. Translate the sentences according to the models.

Model 1. There are various ways of evaluating formulae. – Існують різні способи обчислення формул.

1. There are a lot of important theorems in this book. 2. There are sets containing no elements. 3. There has been recently developed a new method of proving the theorem. 4. There are many measurements to be made. 5. There weren’t any problems with my term paper last year. 6. There will be enough work for everybody at the next conference.

Model 2. There exist a lot of equivalent relations. – Існує багато еквівалентних відношень.

1. There exists no difference between these two expressions. 2. There exists at least

one element in a non-empty set. 3. There exist some important statements in the article. 4. There exist many different ways of defining a circle. 5. There exist no solutions to the problem presented.

Text 6. Vector calculus

Task 1. Read the text about vector calculus

Vector calculus (or **vector analysis**) is a branch of mathematics concerned with differentiation and integration of vector fields, primarily in 3-dimensional Euclidean space R^3. The term "vector calculus" is sometimes used as a synonym for the broader subject of multivariable calculus, which includes vector calculus as well as partial differentiation and multiple integration. Vector calculus plays an important role in differential geometry and in the study of partial differential equations. It is used extensively in physics and engineering, especially in the description of electromagnetic fields, gravitational fields and fluid flow.

Vector calculus was developed from quaternion analysis by J. Willard Gibbs and Oliver Heaviside near the end of the 19th century, and most of the notation and terminology was established by Gibbs and Edwin Bidwell Wilson in their 1901 book, Vector Analysis. In the conventional form using cross products, vector calculus does not generalize to higher dimensions, while the alternative approach of geometric algebra, which uses exterior products does generalize, as discussed below.

Scalar fields. A scalar field associates a scalar value to every point in a space. The scalar may either be a mathematical number or a physical quantity. Examples of scalar fields in applications include the temperature distribution throughout space, the pressure distribution in a fluid, and spin-zero quantum fields, such as the Higgs field. These fields are the subject of scalar field theory.

Vector fields. A vector field is an assignment of a vector to each point in a subset of space. A vector field in the plane, for instance, can be visualized as a collection of arrows with a given magnitude and direction each attached to a point in the plane. Vector fields are often used to model, for example, the speed and direction of a moving fluid throughout space, or the strength and direction of some force, such as the magnetic or gravitational force, as it changes from point to point.

Vectors and pseudovectors. In more advanced treatments, one further distinguishes pseudovector fields and pseudoscalar fields, which are identical to vector fields and scalar fields except that they change sign under an orientation-reversin 126 map: for example, the curl of a vector field is a pseudovector field, and if one reflects a vector field, the curl points in the opposite direction. This distinction is clarified and elaborated in geometric algebra, as described below.

Optimization. For a continuously differentiable function of several real variables, a point P (that is a set of values for the input variables, which is viewed as a point in Rn) is criticalif all of the partial derivatives of the function are zero at P, or, equivalently, if its gradient is zero. The critical values are the values of the function at the critical points. If the function is smooth, or, at least twice continuously differentiable, a critical point may be either a local maximum, a local minimum or a saddle point. The different cases may be distinguished by considering the eigenvalues of the Hessian matrix of second derivatives. By Fermat's theorem, all local maxima and minima of a differentiable function occur at critical points. Therefore, to find the local maxima and minima, it suffices, theoretically, to compute the zeros of the gradient and the eigenvalues of the Hessian matrix at these zeros.

Different 3-manifolds. Vector calculus is initially defined for Euclidean 3-space, which has additional structure beyond simply being a 3-dimensional real vector space, namely: an inner product (the dot product), which gives a notion of length (and hence angle), and an orientation, which gives a notion of left-handed and right-handed. These structures give rise to a volume form, and also the cross product, which is used pervasively in vector calculus. The gradient and divergence require only the inner product, while the curl and the cross product also requires the handedness of the coordinate system to be taken into account (see cross product and handedness for more detail). Vector calculus can be defined on other 3-dimensional real vector spaces if they have an inner product (or more generally a symmetric nondegenerate form) and an orientation; note that this is less data than an isomorphism to Euclidean space, as it does not require a set of coordinates (a frame of reference), which reflects the fact that vector calculus is invariant under rotations (the special orthogonal group SO(3)). More generally, vector calculus can be defined on any 3-dimensional oriented Riemannian manifold, or more generally pseudo-Riemannian manifold. This structure simply means that the tangent space at each point has an inner product (more generally, a symmetric nondegenerate form) and an orientation, or more globally that there is a symmetric nondegenerate metric tensor and an orientation, and works because vector calculus is defined in terms of tangent vectors at each point.

Task 2. Translate the sentences and note the form of the Infinitive.

We consider these two phenomena to be of the same origin. I expect this law to hold for all similar cases. We understand this method to consist of several steps. They wanted us to establish a certain correspondence between these two facts. We assume the program to have been carefully developed. We suppose the particles to be generated at very high speed. We expect this sentence to be true. We know mathematics to have become man’s second language.

We expect a variable or a mathematical expression containing a variable to represent a number. We know two numbers to be relatively prime to each other if their greatest common factor is 1.

We expect this solution to satisfy the given statement. Professor wants his students to attend classes regularly. The students saw their instructor draw (drawing) a line segment. We heard them discuss (discussing) similar questions. Professor wanted his postgraduate students to take part in his research. For a proper correspondence between these phenomena to be established they first have to be considered separately. For correct conclusions to be drawn all the conditions must be observed. It was impossible for the process to continue. I wonder if it is necessary for them to come. For you to begin the work now is very important. For the problem to be understood it must be read carefully.

Task 3. In the following sentences use the Complex Object.

Model: I expect that the article will be written. – I expect the article to be written.

I expect that these rules will be observed. I know that this work is of great importance. He expects that the situation will be analysed carefully. We believe that the machine has certain advantages. I thought she was ready. He expected that I knew the solution. We found that they were interested in the problem. I expect that she will understand me. We expected that he had completed the experiment. I knew that you had obtained similar results. I believed that they had closely cooperated with you. We found that she had studied the material properly. I suppose that he is involved in this discussion. I assume that they have applied the previously obtained data.

Text 7. TOPOLOGY

Task 1. Read the text below.

We know that modern maths is composed of many different divisions. Despite its rigorousness topology is one of the most appealing. Its study is today one of the largest and most important of maths activities.

Although the study of polyhedra held a central place in Greek geometry, it remained for Descartes and Euler to discover the following fact: In a simple polyhedra let V denote the number of vertices, E the number of edges, and F the number of faces; then always $V + F - E = 2$ By a polyhedron is meant a solid, whose face consists of a number of polygonal faces. In the case of regular solids all the polygons are congruent and all the angles at vertices are equal.

A polyhedron is simple if there are no "holes" in it, so that its surface can be deformed continuously into the surface of a sphere. There are of course, simple polyhedra which are not regular and polyhedra which are not simple. It is not difficult to check the fact that Euler's formula holds for simple polyhedra, but does not hold for non simple polyhedra.

We must recall that elementary geometry deals with magnitudes (lengths, angles and areas) that are unchanged by the rigid motions, while projective geometry deals with the concepts (point, line, incidence, and cross – ratio), which are unchanged by the still larger group of projective transformations. But the rigid motions and the projections are both very special cases of what are called topological transformation: a topological transformation of one geometrical figure A into another figure A' is given by any correspondence P ↔ P' between the points P of A and the points P' of A', which has the following two properties: The correspondence is biunique. This means to imply that to each point P of A corresponds just one point P' of A' and conversely.

2. The correspondence is continuous in both directions. This means that if we

take any two point P, Q of A and move P so that the distance between it and Q approaches zero (0), the distance between the corresponding points P', Q' of A' will also approach zero, and conversely.

The most intuitive examples of general topological transformation are deformations. Imagine, a figure such as a sphere or a triangle to be made from, or drawn upon, a thin sheet of rubber, which is then stretched and twisted in any manner without tearing it and without bringing distinct points into actual coincidence. The final position of the figure will then be a topological image of the original.

A triangle can be deformed into any other triangle or into a circle or an ellipse, and hence these figures have exactly the same topological properties. But one cannot deform a circle into a line segment, nor the surface of a sphere into the surface of an inner tube.

The general concept of topological transformation is wider than the concept of deformation. For example, if a figure is cut during a deformation and the edges of the cut sewn together after the deformation in exactly the same way as before, the process still defines a topological transformation of the original figure although it is not a deformation. Topological properties (such as are given by Euler's theorem) are of the greatest interest and importance in many math investigations. There are, in a sense, the deepest and most fundamental of all geometrical properties, since they persist (continue to hold) under the most drastic changes of shape. On the basis of Euler's formula it is easy to show that there are no more than five regular polyhedra.

Task 2. Answer the following questions.

1. Who first discovered the formula V + F – E = 2 for a simple polyhedron?

2. Which solid could be a polyhedron?

3. What happens to the polygons if the polyhedron is regular?

3. Are the magnitudes and projective transformation unchanged in the same cases?

4. What kinds of polyhedra are there?

5. Why can a triangle be deformed into any other triangle, into a circle or into an ellipse?

6. Can we deform a circle into a line segment? Why or why not?

7. How important are topological properties for mathematicians?

Task 3. Are the statements True (T) or False (F)? Correct the false ones.

1. All the angles at vertices of regular polyhedra are even.

2. Any simple polyhedra is regular.

3. Euler's formula holds for any kind of polyhedra.

4. Between the points P of A and the points P' of A' the correspondence is one to one.

5. The correspondence is not interrupted in one direction.

6. Using Euler's formula we can illustrate a lot of regular polyhedra.

Text 8. Zeno

Task 1. Read the text below

There are difficulties in maths concepts of length and time which were first pointed out by the Greek philosopher Zeno, but which can now be resolved by use of Cantor's theory of infinite classes. We've just considered a formulation by Betrand Russell of Zeno's Achilles and the tortoise paradox.

Part of this argument is sound. We must agree that from the start of the race to the end the tortoise passes through as many points as Achilles does, because at each instant of time during which they run each occupies exactly one position. Hence, there is a one–to–one correspondence between the infinite set of points run through by the tortoise and the infinite set of points run through by Achilles.

The assertion that because he must travel a greater distance to win the race Achilles will have to pass through more points than the tortoise is not correct, however, because as we know the number of points on the line segment Achilles must traverse to win the race is the same as the number of points on the line segment the tortoise traverses.

We must notice that the number of points on a line segment has nothing to do with its length. It is Cantor's theory of infinite classes that solves the problems and saves our math theory of space and time. For centuries mathematicians misunderstood the paradox. They though it merely showed its poser Zeno was ignorant that infinite series may have a finite sum. To suppose that Zeno did not recognize it is absurd.

The point of the paradox could not be appreciated until maths passed through the third crisis. Cantor holds that it does make sense to talk of testing an infinity of cases. The paradox is not that Achilles doesn't catch the tortoise, but that he does. In his fight against the infinite divisibility of space and time Zeno proposed other paradoxes that can be answered satisfactorily only in terms of the modern math conceptions of space and time and the theory of infinite classes. Consider an arrow in its flight. At any instant it is in a definite position. At the very next instant, says Zeno, it is in another position. There is no next instant, whereas the argument assumes that there is. Instants follow each other as do numbers of the number system, and just as there is not next larger number after 2 and 1, there is no next instant after a given one.

Between any two instants an infinite number of others intervene. But this explanation merely exchanges one difficulty for another. Before an arrow can get from one position to any nearby position, it must pass through an infinite number of intermediate positions, one position corresponding to each of the infinite intermediate instants. To traverse one unit of length an object must pass through an infinite number of positions but the time required to do this may be no more than one second; for even one second contains an infinite number of instants.

There is, however, a greater difficulty about motion of the arrow. At each instant of its flight the tip of the arrow occupies a definite position. At that instant the arrow cannot move, for an instant has no duration. Hence, at each instant the arrow is at rest. Since this is true at each instant, the moving arrow is always at rest. This paradox is almost startling; it appears to defy logic itself.

The modern theory of infinite sets makes possible an equally startling solution. Motion is a series of rests. Motion is nothing more than a correspondence between positions and instants of time, the positions and the instants each forming an infinite set. At each instant of the interval during which an object is in "motion" it occupies a definite position and may be said to be at rest.

The maths theory of motion should be more satisfying to our intuition for it allows for an infinite number of "rests" in any interval of time. Since this concept of motion also resolves paradoxes, it should be thoroughly acceptable. The basic concept in the study of infinite quantities is that of a collection, a class, or a set of instants in time. Unfortunately, this seeming simple and fundamental concept is beset with difficulties, that revealed themselves in Zeno's paradoxes.

Task 2. **Answer the following questions.**

a. Why are the points passed through by Achilles and the tortoise equal?

b. Did mathematicians at that time agree with Zeno's paradox? What did they think about it?

c. Does it make sense to speak of completing an infinite series of operations in Cantor's opinion?

d. What did Zeno propose to fight against the infinitive divisibility of space and time?

e. Why is not Zeno's explanation satisfactory?

f. Does the maths concept of motion satisfy the conception of physical phenomenon of motion? Why or why not?

g. What is the basic concept in the study of infinite numbers?

Task 3. Find words or phrases from the text that fixed the meaning of the underlined words.

a. At any moment the arrow is in another position.

b. The amount of points on a line segment is not concerned with its length.

c. Between any two instants there is an infinite number of other intermediate ones.

d. The number of points that Achilles and the tortoise passed is equal.

e. At each instant of the period of time during which an object is in motion, it occupies a definite position.

f. Lots of people thought that the paradox is unreasonable.

Text 9. Deduction and Induction

Task 1. Read the text about deduction and induction.

The scientists have proved a chain of theorems and have come to recognize the entire structure of undefined terms, definitions, assumptions, and theorems as constituting an abstract logical system. In such a system we say that each proposition is derived from its predecessor by the process of logical deduction. This process of logical deduction is scientific reasoning. This scientific reasoning must not be confused with the mode of thinking employed by the scientist when he is feeling his way toward a new discovery.

At such times the scientist, curious about the sum of the angles of a triangle, proceeds to measure the angles of a great many triangles very carefully. In every instance he notices that the sum of the three angles is very close to 180°; so he puts forward a guess that this will be true of every triangle he might draw.

This method of deriving a general principle from a limited number of special instances is called induction. The method of induction always leaves the possibility that further measurement and experimentation may necessitate some modification of the general principle.

The method of deduction is not subject to upsets of this sort. When the mathematician is groping for (шукає) new mathematical ideas, he uses induction. On the other hand, when he wishes to link his ideas together into a logical system, he uses deduction. The laboratory scientist also uses deduction when he wishes to order and classify the results of his observations and his inspired guesses and to arrange them all in a logical system.

While building this logical system he must have a pattern (модель) to guide him, an ideal of what a logical system ought to be. The simplest exposition (виклад) of this ideal is to be found in the abstract logical system of demonstrative geometry. It is clear that both deductive and inductive thinking are very useful to the scientist.

Task 2. Find the answers to the following questions.

1. What is logical deduction?
2. Do we proceed from the general to the particular or from the particular to the general in induction?
3. Which method of thinking is more useful: deductive or inductive?

Text 10. Vector field

Task 1. Read the text about vector field.

In vector calculus, a vector field is an assignment of a vector to each point in a subset of space. A vector field in the plane, for instance, can be visualized as a collection of arrows with a given magnitude and direction each attached to a point in the plane. Vector fields are often used to model, for example, the speed and direction of a moving fluid throughout space, or the strength and direction of some force, such as the magnetic or gravitational force, as it changes from point to point.

The elements of differential and integral calculus extend to vector fields in a natural way. When a vector field represents force, the line integral of a vector field represents the work done by a force moving along a path, and under this interpretation conservation of energy is exhibited as a special case of the fundamental theorem of calculus.

Vector fields can usefully be thought of as representing the velocity of a moving flow in space, and this physical intuition leads to notions such as the divergence (which represents the rate of change of volume of a flow) and curl (which represents the rotation of a flow).

In coordinates, a vector field on a domain in n-dimensional Euclidean space can be represented as a vector-valued function that associates an n-tuple of real numbers to each point of the domain. This representation of a vector field depends on the coordinate system, and there is a well-defined transformation law in passing from one coordinate system to the other.

Vector fields are often discussed on open subsets of Euclidean space, but also make sense on other subsets such as surfaces, where they associate an arrow tangent to the surface at each point (a tangent vector). More generally, vector fields are defined on differentiable manifolds, which are spaces that look like Euclidean space on small scales, but may have more complicated structure on larger scales. In this setting, a vector field gives a tangent vector at each point of the manifold (that is, a section of the tangent bundle to the manifold). Vector fields are one kind of tensor field**.**

A vector field for the movement of air on Earth will associate for every point on the surface of the Earth a vector with the wind speed and direction for that point. This can be drawn using arrows to represent the wind; the length (magnitude) of the arrow will be an indication of the wind speed. A "high" on the usual barometric pressure map would then act as a source (arrows pointing away), and a "low" would be a sink (arrows pointing towards), since air tends to move from high pressure areas to low pressure areas.

 Velocity field of a moving fluid. In this case, a velocity vector is associated to each point in the fluid.

 Streamlines, Streaklines and Pathlines are 3 types of lines that can be made from vector fields. They are:

a) streaklines – as revealed in wind tunnels using smoke;

b) streamlines (or fieldlines) – as a line depicting the instantaneous field at a given time;

c) pathlines – showing the path that a given particle (of zero mass) would follow.

* Magnetic fields. The fieldlines can be revealed using small iron filings.

* Maxwell's equations allow us to use a given set of initial conditions to deduce, for every point in Euclidean space, a magnitude and direction for the force experienced by a charged test particle at that point; the resulting vector field is the electromagnetic field.

* A gravitational field generated by any massive object is also a vector field. For example, the gravitational field vectors for a spherically symmetric body would all point towards the sphere's center with the magnitude of the vectors reducing as radial distance from the body increases.

Task 2. Name the Complex Subject and the predicate in every sentence.

Scientists are sure to find a reliable method of detecting errors. The hypothesis proved to be based on the wrong assumption. All the circumstances do not seem to have beenproperly observed.

Certain mistakes appear to have occurred. A proper interpretation of this fact is likely to be obtained. The equipment we were interested in happened to be produced on the line at this factory. Only a century ago the atom was believed to be indivisible. The operator is sure to find errors in the program presented. This question is sure to arise. The computation is expected to have been carried out.

Such a mistake is unlikely to have remained unnoticed. This major occasion is known to have caused a lot of argument. This phenomenon does not seem to obey the general law. This solution is believed to be obviously absurd. The preparatory work proved to be very slow and difficult.

Task 3. Change the sentences according to the model.

Model: It **is believed** that **he** is a reliable business partner.

He is believed to be a reliable business partner.

It is expected that they will detect the error. It is believed that he is very accurate in making calculations. It is known that they have foreseen all the possible mistakes. It is likely that he has given them explicit instructions. It is unlikely that they have supplied this lab with such complex equipment. It appears that they are unable to account for this absurd situation. It seems that he is an intelligent researcher. It happened so that the error was quickly detected.

Glossary of some mathematical terms

Algebra Dealing with letters instead of numbers so as to extend arithmetic, algebra is now a general method applicable to all mathematics and its applications. The word 'algebra' derives from 'al-jabr' used in an Arabic text of the ninth century AD.

Algorithm A mathematical recipe; a set routine for solving a problem.

Argand diagram A visual method for displaying the two-dimensional plane. of complex numbers.

Axiom A statement, for which no justification is sought, that is used to define a system. The term 'postulate' served the same purpose for the Greeks but for them it was a self-evident truth.

Base The basis of a number system. The Babylonians based their number system on 60, while the modern base is 10 (decimal).

Binary number system A number system based on two symbols, 0 and 1, fundamental for computer calculation.

Cardinality The number of objects in a set. The cardinality of the set {a, b, c, d, e} is 5, but cardinality can also be given meaning in the case of infinite sets.

Chaos theory The theory of dynamical systems that appear random but have underlying regularity.

Commutative Multiplication in algebra is commutative if $a \times b = b \times a$, as in ordinary arithmetic (e.g. $2 \times 3 = 3 \times 2$). In many branches of modern algebra this is not the case (e.g. matrix algebra).

Conic section The collective name for the classical family of curves which includes circles, straight lines, ellipses, parabolas and hyperbolas. Each of these curves is found as cross-sections of a cone.

Corollary A minor consequence of a theorem.

Counterexample A single example that disproves a statement. The statement 'All swans are white' is shown to be false by producing a black swan as a counterexample.

Denominator The bottom part of a fraction. In the fraction $\frac{3}{7}$, the number 7 is the denominator.

Differentiation A basic operation in Calculus which produces the derivative or rate of change. For an expression describing how distance depends on time, for example, the derivative represents the velocity. The derivative of the expression for velocity represents acceleration.

Diophantine equation An equation in which solutions have to be whole numbers or perhaps fractions. Named after the Greek mathematician Diophantus of Alexandria (c.AD 250).

Discrete A term used in opposition to 'continuous'. There are gaps between discrete values, such as the gaps between the whole numbers 1, 2, 3, 4, . . .

Distribution The range of probabilities of events that occur in an experiment or situation. For example, the Poisson distribution gives the probabilities of x occurrences of a rare event happening for each value of x.

Divisor A whole number that divides into another whole number exactly. The number 2 is a divisor of 6 because 6 ÷ 2 = 3. So 3 is another because 6 ÷ 3 = 2.

Empty set The set with no objects in it. Traditionally denoted by ê, it is a useful concept in set theory.

Fraction A whole number divided by another, for example $\frac{3}{7}$.

Geometry Dealing with the properties of lines, shapes, and spaces, the subject was formalized in Euclid's Elements in the third century BC. Geometry pervades all of mathematics and has now lost its restricted historical meaning.

Greatest common divisor, gcd T h e gcd of two numbers is the largest number which divides into both exactly. For example, 6 is the gcd of the two numbers 18 and 84.

Hexadecimal system A number system of base 16 based on 16 symbols, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F. It is widely used in computing.

Hypothesis A tentative statement awaiting either proof or disproof. It has the same mathematical status as a conjecture.

Imaginary numbers Numbers involving the 'imaginary' $i = \sqrt{-1}$. They help form the complex numbers when combined with ordinary (or 'real') numbers.

Integration A basic operation in Calculus that measures area. It can be shown to be the inverse operation of differentiation.

Irrational numbers Numbers which cannot be expressed as a fraction (e.g. the square root of 2).

Iteration Starting off with a value a and repeating an operation is called iteration. For example, starting with 3 and repeatedly adding 5 we have the iterated sequence 3, 8, 13, 18, 23, . . .

Lemma A statement proved as a bridge towards proving a major theorem.

Matrix An array of numbers or symbols arranged in a square or rectangle. The arrays can be added together and multiplied and they form an algebraic system.

Numerator The top part of a fraction. In the fraction 3/7, the number 3 is the numerator.

One-to-one correspondence The nature of the relationship when each object in one set corresponds to exactly one object in another set, and vice versa.

Optimum solution Many problems require the best or optimum solution. This may be a solution that minimizes cost or maximizes profit, as occurs in linear programming.

Place-value system The magnitude of a number depends on the position of its digits. In 73, the place value of 7 means '7 tens' and of 3 means '3 units'.

Polyhedron A solid shape with many faces. For example, a tetrahedron has four triangular faces and a cube has six square faces.

Prime number A whole number that has only itself and 1 as divisors. For example, 7 is a prime number but 6 is not (because 6 ÷ 2 = 3). It is customary to begin the prime number sequence with 2.

Pythagoras's theorem If the sides of a right-angled triangle have lengths x, y and z then $x^2 + y^2 = z^2$ where z is the length of the longest side (the hypotenuse) opposite the right angle.

Quaternions Four-dimensional imaginary numbers discovered by W.R. Hamilton.

Rational numbers Numbers that are either whole numbers or fractions.

Remainder If one whole number is divided by another whole number, the number left over is the remainder. The number 17 divided by 3 gives 5 with remainder 2.

Sequence A row (possibly infinite) of numbers or symbols.

Series A row (possibly infinite) of numbers or symbols added together.

Set A collection of objects: for example, the set of some items of furniture could be F = {chair, table, sofa, stool, cupboard}.

Square number The result of multiplying a whole number by itself. The number 9 is a square number because $9 = 3 \times 3$. The square numbers are 1, 4, 9, 16, 25, 36, 49, 64, . . .

Square root The number which, when multiplied by itself, equals a given number. For example, 3 is the square root of 9 because $3 \times 3 = 9$.

Squaring the circle The problem of constructing a square with the same area as that of a given circle – using only a ruler for drawing straight lines and a pair of compasses for drawing circles. It cannot be done.

Symmetry The regularity of a shape. If a shape can be rotated so that it fills its original imprint it is said to have rotational symmetry. A figure has mirror symmetry if its reflection fits its original imprint.

Theorem A term reserved for an established fact of some consequence.

Transcendental number A number that cannot be the solution of an algebraic equation, like $ax^2 + bx + c = 0$ or one where x has an even higher power. The number π is a transcendental number.

Twin primes Two prime numbers separated by at most one number. For example, the twins 11 and 13. It is not known whether there is an infinity of these twins.

Unit fraction Fractions with the top (numerator) equal to 1. The ancient Egyptians partly based their number system on unit fractions.

Venn diagram A pictorial method (balloon diagram) used in set theory.

x–y axes The idea due to René Descartes of plotting points having an xcoordinate (horizontal axis) and y-coordinate (vertical axis).

Appendix 1

Numbers and Arithmetic Operations

/‘nΛmbə(r)/ /ə‘riӨmətik/ /opə‘ræisyen/

Numbers

Two kinds of activity made our ancestors develop numbers (cardinal and ordinal numbers). The first for comparing their things (which one has mor elements), and the second for creating order.

A. Cardinal Numbers (Counting Numbers) /_kɑ: dinl ‘nΛmbə(r)/ /kaunting ‘nΛmbə(r)/

Example:

1 one /wʌn/

2 two /tu:/

3 three /θri:/

4 four /fɔː/

5 five /faɪv/

6 six /sɪks/

7 seven /'sevən/

8 eight /eɪt/

9 nine /naɪn/

10 ten /ten/

11 eleven /ɪ'levən/

12 twelve /twelv/

13 thirteen /θɜ:'ti:n/

14 fourteen /fɔː'ti:n/

15 fifteen /fɪf'ti:n/

16 sixteen /sɪkst'i:n/

17 seventeen /seven'ti:n/

18 eighteen /eɪ'ti:n/

19 nineteen /naɪn'ti:n/

20 twenty /'twentɪ/

21 twenty-one /twentɪ'wʌn/

22 twenty-two /twentɪ'tu:/

23 twenty-three /twentɪ'θri:/

24 twenty-four /twentɪ'fɔː/

25 twenty-five /twentɪ'faɪv/

26 twenty-six /twentɪ'sɪks/

30 thirty /'θɜ:tɪ/

40 forty /'fɔːtɪ/

50 fifty /'fɪftɪ/

60 sixty /'sɪkstɪ/

70 seventy /'sevəntɪ/

80 eighty /'eɪtɪ/

90 ninety /'naɪntɪ/

100 a hundred; one hundred /ə 'hʌndrəd/ /wʌn 'hʌndrəd/

101 a hundred and one /ə 'hʌndrəd ən wʌn/

110 a hundred and ten /ə 'hʌndrəd ən ten/

120 a hundred and twenty /ə 'hʌndrəd ən 'twentɪ/

200 two hundred /tu: 'hʌndrəd/

300 three hundred /θri: 'hʌndrəd/

900 nine hundred /naɪn 'hʌndrəd/

1 000 a thousand, one thousand /ə θ'ɑʊzənd/ /wʌn 'θɑʊzənd/

1 001 a thousand and one /ə 'θɑʊzənd ən wʌn/

1 010 a thousand and ten /ə 'θɑʊzənd ən ten/

1 020 a thousand and twenty /ə 'θɑʊzənd ən 'twentɪ/

1 100 one thousand, one hundred /wʌn 'θɑʊzənd wʌn 'hʌndrəd/

1 101 one thousand, one hundred and one /wʌn 'θɑʊzənd wʌn 'hʌndrəd ən wʌn/

9 999 nine thousand, nine hundred and ninetynine /naɪn 'θɑʊzənd naɪn 'hʌndrəd ən 'naɪntɪ 'naɪn/

10 000 ten thousand /ten 'θɑʊzənd/

15 356 fifteen thousand, three hundred and fifty six /'fɪfti:n 'θɑʊzənd θri: 'hʌndrəd ən 'fɪftɪ sɪks/

100 000 a hundred thousand /ə 'hʌndrəd 'θɑʊzənd/

1 000 000 a million /ə 'mɪljən/

1 000 000 000 a billion /ə 'bɪljən/

1 000 000 000 000 a trillion

B. Ordinal Numbers/Place Numbers /'ərdinəl 'nΛmbə(r)/

Example:

1st first /fɜ:st/

2nd second /'sekənd/

3rd third /θɜ:d/

4th fourth /fɔ:θ/

5th fifth /fɪfθ/

6th sixth /sɪksθ/

7th seventh /'sevənθ/

8th eighth /eɪtθ/

9th ninth /naɪnθ/

10th tenth /tenθ/

11th eleventh /ɪ'levənθ/

12th twelfth /'twelfθ/

13th thirteenth /θɜ:'ti:nθ/

14th fourtheenth /fɔː'ti:nθ/

15th fidteenth /fɪf'ti:nθ/

16th sixteenth /sɪks'ti:nθ/

17th seventeenth /seven'ti:nθ/

18th eighteenth /eɪ'ti:nθ/

19th nineteenth /naɪn'ti:nθ/

20th twentieth /'twentɪəθ/

21st twenty-first /twentɪ'fɜ:st/

22nd twenty-second /twentɪ'sekənd/

23rd twenty-third /twentɪ'θɜ:d/

24th twenty-fourth /twentɪ'fɔ:θ/

25th twenty-fifth /twentɪ'fɪfθ/

26th twenty-sixth /twentɪ'sɪksθ/

27th twenty-seventh /twentɪ'sevənθ/

28th twenty-eighth /twentɪ'eɪtθ/

29th twenty-ninth /twentɪ'naɪnθ/

30th thirtieth /'θɜːtɪəθ/

31st thirty-first /θɜːtɪ'fɜ:st/

40th fortieth /'fɔ:tɪəθ/

50th fiftieth /'fɪftɪəθ/

100th hundredth /'hʌndrədθ/

1 000th thousandth /'θɑʊzəndθ/

1 000 000th miilionth/'mɪljənθ/

Natural Numbers

/‘nætʃral ‘nΛmbə(r)/

1, 2, 3, … one, two, three, and so forth (without end).

1, 2, 3, …, 10 one, two, three, and so forth up to ten.

Natural numbers can be divided into two sets:

Odd Numbers /ɒd ‘nΛmbə(r)/ and Even Numbers /‘i:vn ‘nΛmbə(r)/

Whole Numbers /həʊl ‘nΛmbə(r)/

Natural Numbers + 0 zero/o/nought /‘ziərəu/ /nə:t/

Integers /‘intəjər/

...,--2, 1, 0, 1, ..., negative two, negative one, zero, one, ..

Rational numbers /‘ræʃnəl ‘nΛmbə(r)/ are numbers that can be expressed as fraction.

Irrational Numbers /i‘ræʃnəl ‘nΛmbə(r)/ are numbers that cannot be expressed as fraction, such as √2, π.

Real Numbers /riəl ‘nΛmbə(r)/ are made up of rational and irrational numbers.

Complex Numbers /‘kompleks ‘nΛmbə(r)/

Complex numbers are numbers that contain real and imaginary part.

2 + 3i 2 is called the real part, 3 is called the imaginary part, and it is called imaginary unit of the complex number.

A **Digit** /‘dıdʒıt/ is any one of the ten numerals 0, 1, 2, 3, 4, 5, 6,7, 8, 9.

Example:

3 is a single-digit number, but 234 is a three-digit number.

In 234, 4 is the units digit, 3 is the tens digit, and 2 is hundreds digit.

Consecutive /kən'sekjʊtıv/ **numbers** are counting numbers that differ by 1.

Examples:

83, 84, 85, 86, and 87 are 5 consecutive numbers.

84, 85, 86, … are successor /sək-_ses-ə(r)/ of 83.

84 is the immediate successor of 83.

1, 2, …, and 82 are predecessor /‘predə-ses-ə(r)/of 83.

82 is the immediate predecessor of 83.

36, 38, 40, and 42 are 4 consecutive even numbers.

Operation on Numbers

Addition (+) , Subtraction (-) , Multiplication (×) , Division(:)

/ə‘diʃn/ /sab‘træksyən/ /‘maltəplə‘keisyen/ /di‘vi3n/

Symbols in Numbers Operation

\+ added by/plus/and

/ædid bai/ /plΛs/ /ənd/

\- subtracted by/minus/take away

/səb‘træktid bai/ /‘mainəs/ /teik ə‘wei/

± plus or minus

/plΛs o:(r) _mainəs/

× multiplied by/times

/‘mɅltiplaid bai/ /taimz/

: divided by/over

/di‘vaidid bai/ /‘əuvə(r)/

Symbols for Comparing /kəm‘peə(r)ing / **Numbers**

= is equal to/equals/is

/iz —i:kwəl tu:/ /—i:kwəlz/ /iz/

≠ is not equal to/does not equal

/iz not —i:kwəl tu:/ /‘dɅznt —i:kwəl/

< is less than/is smaller than

/iz les thən/ /iz smõlər thən/

> is greater than/is more than

/iz greitər thən/ /iz mə:(r) thən/

≤ is less than or equal to

/iz les thən o:(r) —i:kwəl tu:/

≥ is more/greater than or equal to

/iz mə:(r)/ /greitər thən o:(r) —i:kwəl tu:/

≡ is approximately equal to

/iz ə‘proksimatli —i:kwəl tu:/

The mathematical sentences that use symbols “=” are called **equation**, and the mathematical sentences that use symbols “<”, “>”, “≤ “, or “≥ “ are called **inequalities.**

Examples

ax + b = 0 is a linear equation.

$ax^2 + bx + c = 0$ is a quadratic equation.

$3x^3 - 2x^2 + 3 = 0$ is a cubic equation.

$\frac{a+b}{2} \geq \sqrt{ab}$ is called AM-GM inequality.

Examples

2 + 3 = 5

two is added by\plus\and three is equal to\equals\is five

2 and 3 are called addends or summands, and 5 is called sum. /sΛm/

10 – 4 = 6

Ten is substracted by\minus\take away four is equal to\equals\is six

10 is the minuend, 4 is the subtrahend, and 6 is the difference/‘difrəns/

7 × 8 = 56

Seven is multiplied by times eight is equal to\equals\is fifty-six

7 is the multiplicator/‘mΛltəplə‘kətwr/,

8 is themultiplicand/‘mΛltəplə‘kənd/, and 56 is the product/‘prodəkt/.

45 : 5 = 9

forty-five is divided by\over five is equal to\is nine

45 is the dividend,

5 is the divisor /də‘vaizə(r)/,

9 is the quotient/‘kwəuʃnt/.

Practice

1. **Read out the following operations, and for every operations name each number‘s function**.

a. 1,209 + 118 = 1,327

b. 135 + (-132) = 3

c. 2 – (-25) = 27

d. 52 – 65 = -13

e. 9 x 26 = 234

f. -111 x 99 = -10,989

g. 36: 9 = 4

h. 1375: (-25) = -55

2. **Fill the blank spaces with the right words**.

a. The _____________ of three and seven is twenty-one.

b. The operation that uses symbol “:” is called ____________.

c. 14 is the __________________________of 13, and the predecessor of 13 are ___________.

d. The result of division is called _______________.

e. Three multiplied ____________ five equals ______________.

f. In 123, 456, 789, the hundred thousands digit is ____, and 9 is the________

g. We select a __________________ number htu, as 100h + 10t + u,

where h represents the ____________ digit, t represents the ______digit, and u represents the units digit.

h. When we ___________ two numbers, for example seven plus thirteen, the answer (twenty) is called ________________.

Fractions /frækʃn/

A common (or simple) fraction is a fraction of the form $\frac{a}{b}$ where a is an integer and b is a counting number

Example: $\frac{p}{q}$

p is called the numerator /nyu:məreitə(r)/of the fraction

q is called the denominator /di'nomi'neitə(r)/ of the fraction

If the numerator < the denominator, then ($\frac{p}{q}$) is a proper fraction /propə(r) _frækʃn/

If the numerator > the denominator, then ($\frac{p}{q}$) is an improper fraction /im'propə(r) _frækʃn/

$3\frac{1}{4}$ is a mixed numbers /miksed 'nΛmbə(r)/ because it contains number part /'nΛmbə(r) pa:t/ and fractional part /_frækʃnəl pa:t/

The fraction $\frac{a}{b}$ is simplified ("in lowest terms") if a and b have no common factor other than 1

Saying Fraction

$\frac{1}{2}$ A/one half /ə/wΛn ha:f/

$\frac{1}{3}$ A/one third /ə/wΛn θɜ:d/

$\frac{1}{4}$ A/one quarter /ə/wΛn _kwɔ:tə(r)/

$\frac{5}{6}$ Five sixths/Five over six

$\frac{(22+x)}{7}$ Twenty-two plus x all over seven

$13\frac{3}{4}$ Thirteen and three quarters

0.3 Nought/zero/o point three

3.056 Three point o five six

273.856 Two hundred and seventy-three point eight five six

Practice

1. Read out the following fractions

a. $\frac{2}{5}$

b. $\frac{3}{4}$

c. $\frac{5}{8}\times\frac{1}{4}=\frac{5}{32}$

d. $2\frac{1}{2} : \frac{9}{10}=3\frac{2}{5}$

e. $\frac{1}{9} - \frac{1}{8}=\frac{1}{24}$

f. 13,945.614

g. 43.554

h. 6.9 x 2.2 = 15.18

i. 72.4 x 61.5 = 4452.6

2. Fill the blank spaces with the right words.

a. In the fraction seven ninths, ___________ is the numerator, and______________ is the _______________.

b. The _______________ of two thirds and a half is four over three.

c. An integer plus a fraction makes a ___________________.

Divisibility

$$\frac{4}{12}$$

12 is divisible by 4./di‘vizəbl/

12 is a multiple of 4 /mΛltipl/

4 divides 12 /di‘vaidz/

4 is a factor of 12/‘fæktə(r)/

15 is not divisible by 4.

If 15 divided by 4 then the quotient is 3 and the remainder is 3 /thə ri‘meındə(r)/ 0 is divisible by all integers

Prime numbers /praim ‘nΛmbə(r)z/

Every numbers is divisible by 1 and itself. These factors (1 and itself) are called improper divisors. /im‘propə(r) də‘vaizə(r)z/

Prime numbers are numbers that have only improper divisors.

Example:

5 is a prime number, but 9 is not a prime number or a composite number. /kompəzit ‘nΛmbə(r)z/

Common Divisors /komən də‘vaizə(r)z/

Example:

1, 2, 3, 4, 6, and 12 are divisors (factors) of 12.

1, 3, 5, and 15 are divisors of 15.

1 and 3 are common divisors of 12 and 15.

3 is the greatest common divisor /greitəst ‗komən də‘vaizə(r)/of 12 and15.

The g.c.d of 12 and 15 is 3.

gcd (12, 15) = 3.

Common Multiples /‗komən ‗mΛltiplz/

Example:

5, 10, 15, 20, 25, …are multiples of 5.

4, 8, 12, 16, 20, 24, … are multiples of 4.

5, 10, 15, 20 are four first multiples of 5.

4, 8,12, 16, 20 are five first multiples of 4.

20, 40, 60, … are common multiples of 4 and 5.

20 is the least common multiple /li:st ‗komən ‗mΛltipl/ of 4 and 5

The l.c.m of 4 and 5 is 20

lcm (4,5) = 20.

Practice

1. Read the following conversation

A: I have two numbers, 36 and 42. Can you say their factors?

B: The factors of 36 are 1, 2, 3, 4, 6, 9,12, 24, and 36. 1, 2, 3, 6, 7, 14, 21, and 42 are factors of 42.

A: So, what are their common factors?

B: They are 1, 2, 3, and 6.

A: And what is the greatest common divisor of 36 and 42?

B: It‘s 6.

2. Make a small conversation about gcd or lcm of other numbers.

Exercise

Write down the spelling of these mathematical sentences

$12 + \frac{1}{3} \leq x - 7$

$3x \times 26 > 20: y$

$x\,(2y + 3) \neq 111.909$

$\frac{(2+x)}{35} < \frac{23}{45}$

Exercise

Use the right words to complete these sentences.

2367 is _______ by nine.

3 is _________ of 34.

The _________ of three and four is twelve.

Eighteen subtracted __ twenty equals _____.

3 is the __________ and 5 is the ________ of three fifths.

Exercise

Write down five first multiples of 8.

Write down all divisors of 18.

Find all common divisors of eighteen and thirty-three.

Write down the simplest form of $\frac{91}{234}$

Find the sum of the reciprocals of two numbers, given that these numbers have a sum of 50 and a product of 25.

What is the product of the greatest common divisor of 9633 and 4693 and the least common multiple of the same numbers?

Let x be the smallest of three positive integers whose products is 720. Find the largest possible value of x.

If P represents the product of all prime numbers less than 1000, what is the value of the units digit of P?

Find a positive integer that is eleven times the sum of its digits?

What is the greatest common divisor of 120 and 49?

The product of 803 and 907 is divided by the sum of 63 and 37. What is the remainder?

The average of four consecutive even integers is 17. Find the largest of the four integers.

When the six-digit number 3456N7 is divided by 8, the remainder is 5. List both possible values of the digit N.

Vocabularies of Chapter I

Words Pronunciation

Numbers /‘nΛmbə(r)z/

Natural Numbers /‘nætʃral ‘nΛmbə(r)z/

Odd Numbers /ɒd ‘nΛmbə(r)z/

Even Numbers /‘i:vn ‘nΛmbə(r)z/

Whole Numbers /həʊl ‘nΛmbə(r)/

Integers /‘intəjərz/

Rational numbers /‘ræʃnəl ‘nΛmbə(r)z/

Irrational Numbers /i‘ræʃnəl ‘nΛmbə(r)z/

Real Numbers /riəl ‘nΛmbə(r)z/

Complex Numbers /‘kompleks ‘nΛmbə(r)z/

Digit /‘dıdʒıt/

Consecutive numbers /kən'sekjʊtıv ‘nΛmbə(r)z/

Prime numbers /praim ‘nΛmbə(r)z/

Composite numbers /kompəzit ‘nΛmbə(r)z/

Addition /ə‘diʃn/

Subtraction /sab‘træksyən/

Multiplication /‘maltəplə‘keisyen/

Division /di‘vi3n/

Equation /ɪ’kweɪʃn/

Inequalities /ˌɪnɪ’kwɒləti/

Difference /‘difrəns/

Sum /sΛm/

Multiplicator /‘mΛltəplə‘kətwr/

Multiplicand /‘mΛltəplə‘kənd/

Product /‘prodəkt/

Dividend /‘dɪvɪdend/

Divisor /də‘vaizə(r)/

Quotient /‘kwəuʃnt/

Fractions /frækʃn/

Numerator /nyu:məreitə(r)/

Denominator /di‘nomi‘neitə(r)/

Proper fraction /propə(r) _frækʃn/

Improper fraction /im‘propə(r) _frækʃn/

Mixed number /miksed ‘nΛmbə(r)/

Numbert part /‘nΛmbə(r) pa:t/

Fractional part /_frækʃnəl pa:t/

II. Powers, Roots, and Logarithm

/'paʊə(r)z/ /ru:tz/ /'lɒgəriðəm/

Powers/Indices /'ɪndɪsi:z/ is used when we want to multiply a number by itself several times.

$$a^b$$

In this term, a is called base/basis /beɪs/'beɪsəs/ and b is called index/exponent /ɪk'spəʊnənt/. The word power sometimes also means the exponent alone rather than the result of an exponential /ɪk'spəʊnənʃl/ expression. How to Say Powers

x^2 x squared /'skweə(r)d/

x^3 x cubed /kju:bd/

x^n x to the power of n

x to the n-th power

x to the n

x to the n-th

x upper /'Λpə(r)/ n

x raised /reizd/ by n

$(x+y)^2$ x plus y all squared

bracket /'brækit/ x plus y bracket closed squared

x plus y in bracket squared

Practice

A. Read out the following terms and say their values.

1. 2^6
2. $(\frac{2}{3})$ 3
3. $x^5 : x^2$
4. $(3ab)^4$
5. $(9x)^0$

LAWS FOR POWERS

for equal exponents

First Law for Power:

$(ab)^n = a^n\ b^n$

A product raised by an exponent is equal to product of factors raised by same exponent

$(\frac{a}{b})^n = a^n/b^n$

For equal basis

Second Law for Powers:

$a^m\ (xa)^n = a^{(m+n)}$

The product of two powers with equal basis equals to the basis raised to the sum of the two exponents

When expressions with the same base are multiplied, the indices are added

How can we say this rule?

a^m: a^n= $a^{(m-n)}$

Third Law for Powers:

$(a^{m)n}$ = a^{mn}

Practice

Try to express in words these another rules of powers:

a^o=1, a≠0

a^{-n} = $\frac{1}{an}$, a≠0

($\frac{a}{b}$)^n= a^n/b^n

a^m: a^n= $a^{(m-n)}$

Roots and Radicals /rædıklz/

Root is inversion of exponentiation

$\sqrt[n]{a}$=b↔ b^n=a

$\sqrt[n]{a}$ is called radical expression (or radical form) because it contains a root.

The radical expression has several parts:

the radical sign /saın/ √

the radicand /rædıkən/: the entire quantity under the radical sign

the index: the number that indicates the root that is being taken example:

∛(a+b) a + b is the radicand, 3 is the index.

The radical expression can be written in exponential form (powers with fractional exponents)

example: $\sqrt[n]{x}$ = $x^{(\frac{1}{n})}$

So the law of powers can be used in calculating root

Examples:

$$\sqrt[n]{ab} = (ab)^{\frac{1}{n}} = a^{\frac{1}{n}}b^{\frac{1}{n}} = \sqrt[n]{a}\sqrt[n]{b}$$

A number is said perfect square if its roots are integers.

Example:

9, 16, 36, and 100 are perfect squares, but 12 and 20 are not.

How to Say Radicals

How to Say Radicals

$\sqrt{x}$	**(square) root of x**
$\sqrt[3]{y}$	**cube root of y**
$\sqrt[n]{z}$	**n-th root of z**
$\sqrt[5]{x^2}\, y^3$	**fifth root of (pause) x squared times y cubed** **fifth root of x squared times y cubed in bracket**

Square Root

The square root is in simplest form if:

a. the radicand does not contain perfect squares other than 1.

b. no fraction is contained in radicand.

c. no radicals appear in the denominator of a fraction.

Example

$\sqrt{24}$ is not a simplest form because we can write it as $\sqrt{(4\times6)}$ where 4 is a perfect square. We can simplify the radical into $2\sqrt{6}$

A radical and a number is called a binomial /baɪ'nəʊmɪəl/. The conjugate /'kɒndʒʊgeɪt/ of binomial is another binomial with the same number and radical, but the sign of second term is changed.

Example $2 + \sqrt{6}$ is a binomial and its conjugate is $2 - \sqrt{6}$

Practice

a. Read out the following radical expressions and say theirs exponential notation.

1. $\sqrt{4x^4}$
2. $\sqrt[4]{m^3}\ n^8$
3. $\sqrt[5]{a^3}$

4. $\sqrt[3]{8x^6}\ y^9$
5. $\sqrt{x^2} + y^2$

b. Read out the following terms and say what their values are:

1. $243^{\frac{1}{5}}$

2. -4^{-2}

3. $125^{\frac{1}{3}}$

Активация Windc
Чтобы активировать V
перейдите в раздел "Г

4. $(-5)^{-1}$

5. 3^{-3}

c. Simplify these radicals

1. $\sqrt{72}$

2. $\sqrt{234}$

3. $\frac{5}{2+\sqrt{3}}$

4. $\frac{\sqrt{3}}{\sqrt{6}-\sqrt{2}}$

d. Find the conjugate of these binomials

1. $2 + \sqrt{5}$

2. $6 - \sqrt{4}$

Logarithm

$x = a^b \leftrightarrow b = a_{\log x}$

In this term, a is also called **base**.

How to Say Logarithm

$n_{log}\, x$	**log /lɒg/x to the base of n** **log base n of x**
ln 2	**natural log of two** **“L N” of two**
$5_{log^2}\, 25$	**log squared of twenty-five to the base of five** **log base five of twenty-five all squared**

Practice

Read out the following terms:

a. $a^x \log b$

b. $\log a^2$

c. $2_{log}\, (\frac{1}{6})$

d. $5_{log}\, (x^2 + y)$

e. $(n_{log} x)^2$

f. $6_{log^2}\, 22 - 6_{log}\, x^2 - 1$

Laws for Logarithm

First Law for logarithm:

The logarithm of a product is equal to the sum of the logarithm of the factors b log(xy)= b log x+ b log y

Second Law for logarithm:

The logarithm of a quotient is equal to the difference of the logarithms of the dividend and divisor

b log(x/y)=b log x - b log y

Third Law for logarithm:

The logarithm of a power is equal to the exponent times the logarithm of the basis

b $\log(x)^2$= a b log x

More Examples

$\mathbf{2_{log}\ (x+y) + 2\ 2_{log}}$ **4x>4**	log base two of x plus y in bracket plus two times log base two of four x's is greater than four
$\boldsymbol{x^2 + \frac{1}{\sqrt{x}} = 1}$	x squared plus (pause) one over root of x equals one
$\mathbf{3^x + 9^{x-1} > 27}$	three upper x plus (pause) nine upper x minus one (pause) is more than twentyseven
$\mathbf{9^x - 1 < 2}$	nine to the x (pause) minus one is less than two

Some Algebraic Processes

1. Expand (x-3) (x+2) into $x^2 - x - 6$
2. Simplify (2x+2)/(x+1) into 2
3. Factorize $x^3 - 2x^2 + 3x - 2$ into (x-1) (x+1) (x-2)
4. Cancel (x+1) from (2x+2)/(x+1) to get 2
5. Add/subtract/multiply/divide both side

Examples: multiply both side of equation ½ x= 4 with 2 to get x=8

6. Subtitute y=4 into equation 2x+y=12
7. Collect (x+2) from $(x+2)^3$-2(x+2) (x+1) to get (x+2) $[(x+2^2) - 2(x+1)]$

Example

Find x that satisfy equation $3^x - 3^{x-1} = 162$.

Answer

First, we multiply both side with 3 to get $3.3^x - 3^x = 486$.

Then, we collect 3^x and we have $3^x(3-1) = 486$, which can be simplified into 2.3^x=486.

Divide both side by 2, we get 3^x=243.

We know that 243 is 3^5, so we can write 3^x=3^5.

According to the rule of powers, x must be equal to 5.

Exercise

How do we say these mathematical terms?

1. $x^{n\ \log(x+1)} = 0$
2. $\sqrt{2\sqrt{2}} = log\left(\frac{x}{5}\right)$

Read the complete answers.

1. $13^2 = \cdots$
2. $2^9 = \cdots$
3. Every positive real numbers has .... real-numbered square roots.
4. The cube root of two hundred and sixteen is ………….
5. If the root of eighty-one is raised by three, then we have ……….
6. 7 is the log base ten of ………….

Solve this problem and try to explain it.

1. Which is greater, $2^{95} + 2^{95}$ or 2^{100}?
2. Which is the larger, $10^{\frac{1}{10}}$ or $2^{\frac{1}{3}}$?

Активация Wi

In March, the number of students was a perfect square. At the end of the semester, with 100 new students, the number of students became 1 more than a perfect square. At the end of the year with an additional 100 new students, the number of students is a perfect square. How many students were there in September?

Explain the process to find the solution x that satisfies each equation, inequality, or system of equations.

1. $1 - 4x \le x + 11$

2. $\frac{3}{2}y - \frac{5}{3} = \frac{4-2y}{5}$

3. $x + y = 2$

$2x - y = -5$

Vocabularies of Chapter II

Words Pronunciation

Powers /'paʊə(r)z/

Indices /‘nætʃral ‘nΛmbə(r)z/

Base /beɪs/

Basis /'beɪsəs/

Exponent /ɪk'spəʊnənt/

Roots /ru:tz/

Radicals /rædɪklz/

Radical sign /rædɪklz saɪn/

Radicand /rædɪkən/

Perfect square /‘pɜ:fɪkt skweə(r)/

Binomial /baɪ'nəʊmɪəl/

Conjugate /'kɒndʒʊgeɪt/

Logarithm /'lɒgəriðəm/

Expand / ɪk‘spænd/

Simplify /‘sɪmplɪfaɪ/

Factorize /‘fæktəraɪz/

Cancel /‘kænsl/

Substitute /‘sΛbstɪtju:t/

Side /saɪd/

Left-hand side /left hænd saɪd/

Right-hand side /raɪt hænd saɪd/

Collect /kə‘ləkt/

Eliminate /ɪ’lɪmɪneɪt/

III. Sequence, Series, and Trigonometry

/'si:kwəns/ /'sɪəri:z/ /trɪgə'nɒmətrɪ/

Arithmetic /ə'rɪθmətɪk/ Sequence/Progression /'prə'greʃn/

An arithmetic sequence is a sequence of the form a, a+d, a+2d, …

The number a is the first term /tɜ:m/, and d is the common difference /'kɒmən 'dɪfrəns/ of the sequence. The difference between two consecutive /kən'sekjʊtɪv/ terms is d

$$a_n - a_{n-1} = d$$

The nth Term of an Arithmetic Sequence

The **nth term** of the arithmetic sequence a, a+d, a+2d, … is

a + (n-1) d

a_1, a_2, a_3 are the **first three terms.**

$a_{n+1/2}$ is called **middle** /'mɪdl/ **term.**

a_n and a_{n+1} are **two consecutive terms**

Partial /'pɑ:ʃl/ sum of Arithmetic Sequence

For arithmetic sequence a, a+d, a+2d, …, the nth partial sum

Sn = (n/2) (2a + (n − 1) d)

or Sn = (n/2) (a + an)

Examples

In arithmetic sequence 1,4,7,10, 13, ..., the first term is 1, the difference is 3. So, the formula for n-th term is 1+(n-1)3=3n-2. Applying the formula we can say the 100-th term of the sequence is 298. The partial sum of the sequence is (n/2) (1+3n-2) = (3/2) n^2-(n/2).

Geometric /dʒɪə'mətrɪk/ **Sequence**

a, ar, ar^2, … is a geometric sequence

a_1= a is the first term.

$a_n = ar^{n-1}$ is the **n-th term** of the geometric sequence.

r is the **common ratio** /'reɪʃɪəʊ/.

Partial Sum of Geometric Sequence

Let Sn be the partial sum of the geometric sequence. Then

Sn = a + ar + $ar^2 + \ldots + ar^{n-1}$

or Sn = $a\frac{|r^n-1|}{r-1}$

Examples

In arithmetic sequence 2,4,8,16, 32, ..., the first term is 2, the ratio is

also 2. So, the formula for n-th term is 2 $2^{n-1} = 2^n$ Applying the formula we can

say the 10-th term of the sequence is 1024. The partial sum of the sequence is

$2\,\frac{(2^n-1)}{(2-1)} = 2^{n+1} - 2.$

The sum of finite /'faɪnaɪt/or infinite /'ɪnfɪnət/ sequence

$$\sum a_n = a_1 + a_2 + \cdots$$

is called **series.**

Practice

Find the first 6 terms and the 300th term of the arithmetic sequence 13,7, ...

The 10th term of an arithmetic sequence is 55 and the 2nd term is 7. Find the 1st term.

Find the sum of the first 40 terms of the arithmetic sequence 3, 7, 11, 15, ...

Find the 8th term of the geometric sequence 5, 15, 45, ...

The 3rd term of a geometric sequence is $\frac{63}{4}$, and the 6th term is $\frac{1701}{32}$. Find the 8th term.

Find the sum of the first 5 terms of the geometric sequence 1, 0.7, 0.49, 0.343, ...

Appendix 2

Writing a summary

A summary is a shorter version of the original text. Such a simplification highlights the major points from the much longer subject, such as a text, speech, film, or event. In order to write a good summary, use your own words to express briefly the main idea and relevant details of the piece you have read. Your purpose in writing the summary is to give the basic ideas of the original reading. 5 main steps in writing summary:

1. Look through the text and try to divide the text into parts. Determine what type of text you are dealing with. Don’t take any notes - just read.

2. Read the text, highlight important information and take notes.

3. Write down the main points of each part, in your own words (this will make it easier to write later).

4. Write down the key support points for the main topic, but do not include minor detail.

5. Go through the text again, make changes as appropriate.

Useful language

Introduction (Title) is a novel by (author). (Title) was written by (author). The story is about (topic). The novel tells the story of (hero/topic). (Title) tells of (hero), who ... In (title) by (author), the reader is taken into (place/time of story). (Title) is the story of (hero/action/...) (Title) is set in the period of (event). The text presents/describes…

Content

As the story begins, ... During ... While ... As/When ... Since/As ... Just then ... After ... Before ... Before long ... Soon ... Soon afterwards ... As soon as ... One day/evening ... The following day ... Some time later ... Hours/Months/Years later, ... By morning/the next day/the time ... Meanwhile ... However, ...

Again/Once again ... At this point ... To his surprise ... This incident is/was followed by ... To make matters even worse ... Eventually, .../Finally, ... The author Says, states, points out that… Claims, thinks, believes that… Describes, explains, makes clear that… Criticizes, analyses, comments on…

Tries to express… Argues that… Suggests that… Compares X to Y… Doubts that… Tries to convince the readers that… Concludes that…

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НАВЧАЛЬНО-МЕТОДИЧНЕ ВИДАННЯ

Укладачі: Капітан Т. А., Лелека Т. О.

ENGLISH FOR MATHEMATICIANS

Навчально-методичний посібник
з курсу «Іноземна мова (англійська) за професійним спрямуванням»
для студентів факультету математики, природничих наук та технологій
Спеціальність 014.04 Середня освіта (Математика)
Освітня програма Середня освіта (Математика та Інформатика)
Середня освіта (Математика, Економіка).

Формат 60х84 1 /16.
Папір офсет. Цифровий друк.
Ум. др. арк. 14. Тираж 100.

Віддруковано: ФОП Піскова М.А.
м. Кропивницький, вул. Тараса Карпи, 17-11.
СВІДОЦТВО ПРО ДЕРЖАВНУ РЕЄСТРАЦІЮ
№ 24440000000027816 від 18.08.2016.